Unemployment Insurance in Macroeconomic Stabilization with Imperfect Expectations*

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Preliminary – comments welcome!

Abstract

We study the power of state-dependent unemployment insurance (UI) to stabilize short-run fluctuations, allowing for arbitrary deviations from full information and rational expectations. Expectations are critical because higher UI generosity raises consumption, to a large extent, by lowering precautionary savings. If UI generosity is indexed to the unemployment rate, households must forecast the unemployment rate to anticipate the policy stance. We estimate unemployment expectations in response to identified aggregate shocks. We quantify the consequences of these imperfect expectations through the lens of a Heterogeneous Agent New Keynesian model. First, we work directly with the estimated forecast errors. Our methodological contribution is to use the non-parametric history of forecast errors and forecast revisions to solve dynamic decisions of optimizing agents. By doing so, we sidestep the need to choose a particular model of belief formation. The estimated model implies that imperfect anticipation substantially affects the stimulative power of UI extensions. Second, we compare alternative ways of implementing UI policies. To run counterfactuals, we estimate a structural model of belief formation. We show that a combination of noisy information and diagnostic expectations fits the data best among a large set of popular alternatives. A UI extension that is announced directly is more stimulative in the very short run than one that is indexed to the unemployment rate.

Keywords: information frictions, bounded rationality, expectations surveys, unemployment insurance, heterogeneous agent New Keynesian models, stabilization policy.

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1 Introduction

Unemployment insurance (UI) is an essential component of the social safety net. Temporary UI duration extensions are among the most commonly used fiscal-policy instruments to fight recessions. In the U.S., legislators have passed additional extensions on five separate occasions in the last 40 years. For example, during the Great Recession, the maximum duration of UI benefits increased from 26 weeks to 99 weeks. More recently, during the 2020 recession, unemployment benefits were again extended by 13 weeks. Despite the ubiquitous nature of UI extensions, their benefits and costs remain debated.

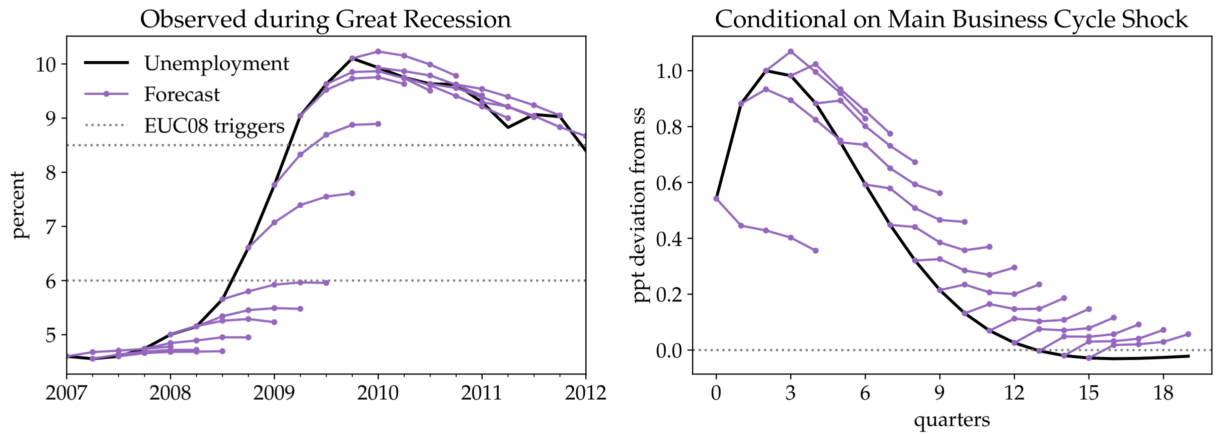
A recent literature emphasizes that a central channel by which UI operates is the households' *precautionary saving* motive (e.g., McKay and Reis 2016 and Kekre 2021). An increase in UI generosity boosts aggregate demand by reducing households' incentives to save in anticipation of unemployment spells. However, this modern literature assumes that people have full-information and rational expectations (FIRE). Assuming FIRE is consequential since precautionary saving depends on people's expectations regarding unemployment risk.

It is now well documented that survey data on beliefs show large deviations from FIRE (e.g., Coibion and Gorodnichenko 2012, 2015, Bordalo, Gennaioli, Ma and Shleifer 2020). For illustration, the left panel in Figure 1 shows the unemployment rate during the Great Recession alongside the consensus forecast for this variable at multiple horizons in the Survey of Professional Forecasters (SPF).[1] We highlight two main facts. First, SPF beliefs systematically under-forecasted the increase in the unemployment rate during the buildup phase. Second, following the peak of unemployment, forecasts lagged behind the decline in actual unemployment. To see why such mistakes may be relevant, note that Emergency Unemployment Compensation 2008 (EUC08) stipulated that UI benefits would be increased by an additional 13 weeks in case the unemployment rate increased above 6 percent. At the national level, this unemployment rate is reached in the third quarter of 2008. Interestingly, right until the quarter just before that, professional forecasters did not anticipate that the unemployment rate would ever cross the 6 percent threshold. This suggests that people may not have expected that Tier 3 would be activated.

When people must forecast the unemployment rate to infer UI generosity, expectations become critical to the success of UI extensions. In this paper, we are interested in understanding the power of UI extensions to stabilize business cycle fluctuations when people's expectations are taken directly from data, as opposed to forcing expectations to be FIRE.

[1] This figure does not represent definite proof for the failure of FIRE because new shocks could be realized at every point in time. The right panel shows that the same pattern of initial under-reaction followd by over-reaction is also present in the impulse responses of beliefs to an identified shock–the main business cycle shock of Angeletos, Collard and Dellas (2020).

Figure 1: Consensus Forecast of Unemployment Rate



Notes. On both panels, the black line is based on the seasonally-adjusted civilian unemployment rate from the U.S. Bureau of Labor Statistics, and the purple line is based on the median unemployment forecast from the Survey of Professional Forcasters. On the left panel, we plot the level of the unemployment rate and forecasts between 2007Q1 and 2012Q1. On the right panel, we plot the impulse responses of these variables to the main business cycle shock (targeting unemployment) from Angeletos, Collard and Dellas (2020). We estimate the impulse responses using an ARMA-IV specification as in Angeletos, Huo and Sastry (2021). We scale the shock such that peak response of unemployment is 1 percentage point.

Illustrative model. We begin our analysis with an illustrative model that isolates the role of expectations in determining the equilibrium consumption response to UI benefit extensions. We work with a two-period setting which allows for an analytical solution. In both periods, workers can be either employed, earning labor income, or unemployed, earning UI benefits. An individual's unemployment shock is independent across periods. We consider a demand-induced recession in the second period, which increases households' precautionary saving motive in the first period. We compare two UI extension policies, that give rise to different expectations in period 0. The first is a policy rule that indexes unemployment benefits to the unemployment rate. The second is a policy of directly announcing unemployment benefits. We show that the difference between the responses of output in these two economies is given by the product of four terms:

$$dY_0^{\text{rule}} - dY_0^{\text{ann}} = -\mathcal{M} \cdot M_b \cdot \zeta_b \cdot \underbrace{(1-\lambda)dU_1}_{\text{forecast error}}, \tag{1}$$

where each term is: (1) the Keynesian-cross multiplier, $\mathcal{M} > 0$, (2) the partial-equilibrium response of aggregate demand to higher anticipated unemployment benefits, $M_b > 0$, (3) the elasticity of unemployment benefits to the unemployment rate, $\zeta_b > 0$, and finally (4) the forecast error in predicting the unemployment rate $(1-\lambda)dU_1$ where $1-\lambda$ denotes the *cognitive bias* and it is such that $dU_1^e = \lambda dU_1$, and $dU_1 > 0$ the increase in unemployment at time 1.

The relative performance of rules-based policy depends on whether beliefs under-react relative to FIRE ($\lambda < 1$) or over-react relative to FIRE ($\lambda > 1$). If individuals have FIRE, $\lambda = 1$, the output response is the same in both scenarios. Instead, if beliefs under-react compared to FIRE, individuals under-forecast the increase in UI benefits. It follows that the stabilization power of the policy is weaker. Instead, if beliefs over-react, the opposite happens. Individuals over-forecast future UI benefits, leading to a larger cut in precautionary savings and thus a milder recession. This model emphasizes that the anticipation of unemployment benefits is an important margin by which these policies transmit to consumption.

General framework. The simple model emphasizes the importance of getting expectations right in assessing the effects of UI extensions. To quantify the consequences of the empirical patterns of expectations, we provide a general framework that allows a more complete description of the economy and its actors and a more general description of their beliefs.

In section 3, we discuss a method to solve dynamic macroeconomic models under arbitrary beliefs about macroeconomic outcomes. This flexible method is based on the Sequence-Space Jacobian (SSJ) framework developed in Auclert, Bardóczy, Rognlie and Straub (2021) and extended to deviations from FIRE by Auclert, Rognlie and Straub (2020).[2] Building on their insights, we show that, to solve for aggregates, it is sufficient to describe how people respond to two additional objects: *forecast errors* and *forecast revisions*.

In the SSJ framework, FIRE is equivalent to perfect foresight. So, people make no forecast errors or revisions. It suffices to describe how people respond to the time-0 innovation, which forces the economy to deviate from a steady state. The Jacobians are sufficient statistics mapping changes in the path of endogenous and exogenous variables into the path of aggregate decisions of the agent block. For example, the consumption-real-interest-rate Jacobian $\mathcal{J}^{C,r}$ maps the change in real interest rates to the change in aggregate consumption of a household block. But, with general beliefs, people make mistakes in forecasting and may revise their expectations in the future. How individuals respond to these new objects can be computed directly from the FIRE Jacobian. The intuition for this result follows from the fact that because forecast errors and revisions are entirely unanticipated by the agents, then their response to these forecast updates is the same as their response to an unanticipated time-0 change. For example, the response of the household block to a forecast error in the time 1 real interest rate $r_1 - r_1^e$ is precisely the same as the agent would respond to a time 0 real interest rate shock r_0 under perfect foresight, $\mathcal{J}^{C,r}_{0,0}$.

Because it only uses the FIRE Jacobians, this method is very fast and easy to implement. We discuss how to implement a variety of popular models of deviations from FIRE.[3] More importantly, the framework allows us to work with arbitrary expectations. We leverage this fact by working directly with empirically measured expectations. This allows us to quantify the impact

[2] Auclert, Rognlie and Straub (2020) show how to manipulate the FIRE Jacobian to implement sticky expectations (Mankiw and Reis 2002, Carroll, Crawley, Slacalek, Tokuoka and White 2018), cognitive discounting (Gabaix 2020), and dispersed information (Angeletos and Huo 2021). They also discuss how this idea can be extended to other models.

[3] See Appendix B.3.

of imperfect expectations on the power of UI extensions in stimulating aggregate demand without imposing any extra assumptions on the belief formation model. Effectively, this method allows us to sidestep the issue of choosing among the "wilderness" of alternative models of belief formation, a common criticism of this literature going back to Sims (1980) and Sargent (2001).

Quantitative framework and results. Equipped with a framework to solve and analyze dynamic models with arbitrary deviations from FIRE, we refine the analytical results in (1). We need four objects. First, the dynamics of forecast errors and revisions about the unemployment rate. Second, a UI extension policy that indexes duration to the unemployment rate. Third, a model of households that maps beliefs about UI duration into aggregate demand. Fourth, a model of the macroeconomy that maps changes in aggregate demand into equilibrium outcomes.

To obtain empirically relevant forecast errors, we estimate the impulse responses of the unemployment rate and its forecasts at different horizons to an identified aggregate shock. We measure expectations as the consensus forecast from the SPF. Our identified shock is the main business cycle (MBC) shock of Angeletos, Collard and Dellas (2020). The MBC shock is a natural choice because it accounts for the largest share of unemployment fluctuations over the business cycle by construction.

We implement automatic UI extensions via a policy rule that indexes the UI expiration probability to the equilibrium unemployment rate. We calibrate the semi-elasticity in the rule, ζ_b, to match the ratio of UI extensions in the EUC08 policy to the rise in the unemployment rate during the Great Recession. The calibrated rule implies that a one percentage point increase in the unemployment rate triggers about a one-quarter increase in average UI duration.

We embed this policy rule in a New Keynesian model with incomplete markets, heterogeneous households, and search and matching frictions. Our model incorporates many features that have been emphasized in modern models of social insurance (McKay and Reis, 2016, Kekre, 2021). Notably, it features intertemporal optimization by risk-averse, borrowing-constrained households; heterogeneity in marginal propensities to consume (MPC); endogenous unemployment risk; and nominal rigidities. We estimate our model following the procedure popularized by Christiano, Eichenbaum and Evans (2005) and recently extended to an heterogeneous-agents environment by Auclert, Rognlie and Straub (2020). First, we calibrate the model's steady state to match a list of relevant moments, including MPCs. Second, we estimate the remaining structural parameters by matching the empirical impulse responses of select aggregate variables. Importantly, this estimation exercise can be performed using directly the expectations observed in the data in response to the identified shocks.

Our estimated model implies that perceived UI duration is more important for aggregate stabilization than actual UI duration. UI extensions raise income only for those workers who experience a job loss and stay eligible thanks to the extension. Most households remain employed even in deep recessions; and so, for them, only perceived UI duration matters, which affects their precautionary saving. So expectations are crucial in assessing the effectiveness of the policy. We show that the policy is less effective in the short run relative to FIRE benchmark. This finding is a direct consequence of the initial under-reaction observed in Figure 1. However, after the peak of the recession, expectations turn overly pessimistic. This pattern of delayed over-reaction implies that the policy becomes even more effective under the estimated beliefs than under FIRE.

We use our model to quantify the impact of UI extensions on equilibrium unemployment and consumption relative to a counterfactual scenario in which UI duration was constant. In order to run counterfactuals, we describe and estimate a model of belief formation which combines noisy information with long-memory diagnostic expectations. We show that, at the onset of the recession, the policy reduces the unemployment rate by 0.4 percentage points and increases aggregate consumption by 0.6 percentage points, while in FIRE the same policy would have reduced the unemployment rate by 0.7 percentage points and increased consumption by 1 percentage point. It follows that the initial belief under-reaction makes this type of policy almost half as effective as would be predicted by models with full-information and rational expectations. However, due to the pattern of delayed over-reaction, the impact of the policy on aggregates is hump-shaped in our model (instead, with FIRE, the peak effectiveness happens immediately). In our model, the peak effectiveness of the policy leads to a reduction of 0.5 percentage points in unemployment and an increase in consumption of almost 1 percentage point.

Finally, we use our model to assess the relative efficacy of different forms of policy communication. In particular, as in Bianchi-Vimercati, Eichenbaum and Guerreiro (2021), we evaluate the stabilization power of announcing the UI duration directly to people rather than implementing as a contingent rule. We conclude that announcing the policy directly can be very stimulative in the very short run, but may lack efficacy later in the recession as expectations turn overly pessimistic.

Relationship to the literature. Our paper contributes to an extensive literature analyzing the consequences of macroeconomic shocks and policies without FIRE and exploiting survey data to calibrate the expectational components of macro models. We share the interest in analyzing these questions in the context of Heterogeneous-agent New Keynesian models (HANK) with the recent contributions by Farhi and Werning (2019), Farhi, Petri and Werning (2020), Auclert, Rognlie and Straub (2020), Pappa, Ravn and Sterk (2023), Dobrew, Gerke, Giesen and Röttger (2023), and Guerreiro (2022). These papers consider parametric models of bounded rationality. We deviate from their contributions in two ways. First, we study the effects of UI extensions on the economy. Second, we discuss a method that allows us to quantify the impact of deviations from FIRE in a non-parametric way, directly exploiting the data coming from surveys of expectations. This allows us to sidestep the discussion of choosing a particular model of deviation from FIRE.

The idea that unemployment expectations are important for business cycles goes back to Carroll (1992). Our model includes a list of features (incomplete markets, nominal rigidities, and suboptimal monetary policy) which have been found important in this line of research since then. Christiano, Eichenbaum and Trabandt (2016) show that nominal rigidities and constraints on monetary policy adjustment tend to reverse the contractionary effects of UI extensions in Krusell, Mukoyama and Şahin (2010), Nakajima (2012), and Mitman and Rabinovich (2015, 2019). Furthermore, Kekre (2021) emphasizes how these mechanisms can be complemented by the stimulus effect arising from the direct redistribution across workers with different marginal propensities to consume and the impact of reducing precautionary savings motives. Relatedly, Bilbiie, Primiceri and Tambalotti (2022) find that cyclical income risk and precautionary saving behavior substantially amplify business cycles. However, this literature has worked exclusively with FIRE. Our paper contributes a new perspective on the quantitative relevance of the different mechanisms when beliefs accord to the survey evidence.

In a closely related paper, Fernandes and Rigato (2022) study UI in a model where households have present-biased preferences. Present bias reduces the responsiveness of precautionary saving to UI extensions. However, they maintain the assumption of full-information rational expectations, making their contribution complementary to ours.

Outline. The structure of the paper is as follows. Section 2, analytical model. Section 3, general framework with propositions. Section 4, quantitative model and results. Section 5 concludes.

2 Illustrative model

We start with an analytical demonstration that imperfect expectations interfere with the power of unemployment insurance (UI) extensions to stabilize business cycles. We consider a simple two-period environment. We engineer a recession in period 1, which triggers precautionary responses in period 0. Then, we analyze how equilibrium output at time 0 depends on households' expectations and the implementation of UI. Appendix A contains detailed derivations and proofs.

2.1 Setup

Consider a two-period model, $t = 0, 1$. The economy is populated by a measure one of households, a representative firm, and a government. The sequence of events within the two periods is the same. First, the representative firm randomly hires a fraction of households. Second, production takes place and households make a consumption-saving decision.

Firm. A competitive firm produces a final good Y_t from labor N_t according to the production function

$$Y_t = N_t \tag{2}$$

The only cost of production is the real wage bill $w_t N_t$ paid to workers. In equilibrium, $w_t = 1$ and the firm hires just enough workers to meet aggregate demand while making zero profit.

Households. In period t, a fraction $N_t \in [0, 1]$ of households is employed. The remaining $1 - N_t$ households are unemployed. The probability that an individual household is employed is the same for all workers and equal to the employment rate N_t. Employed workers earn real wage $w_t = 1$. Unemployed workers receive real benefits $b_t \in (0,1)$, financed by a lump-sum tax τ_t levied on all households. Once their employment status for the current period ($e_t \in \{0,1\}$) is determined, households choose consumption c_t and savings a_t in a non-contingent bond with real return r to maximize their anticipated life-time utility

$$u(c_0) + \beta u(c_1) \tag{3}$$

subject to period budget constraints

$$c_t + a_t = (1+r)a_{t-1} + e_t w_t + (1-e_t)b_t - \tau_t \tag{4}$$

and borrowing constraints $a_t \geq 0$ for $t = 0, 1$. Let the felicity function $u(\cdot)$ be smooth, increasing, concave, and have a positive third derivative, i.e. households are prudent in the sense of Kimball (1990).

At time 0, households may not have perfect foresight of the endogenous variables N_1, b_1, τ_1, and hence even of their own consumption c_1^e. Let N_1^e, b_1^e, τ_1^e, and c_1^e denote the beliefs for each variable. We assume that all households have the same beliefs and do not consider uncertainty regarding the beliefs. We focus on the first-order response of this economy around a non-stochastic equilibrium. Adding uncertainty would not change our results.

Prudence and market incompleteness implies that households have precautionary saving motive in period 0 against unemployment risk in period 1. We can see this from the Euler equation

$$u'(c_0) \geq \beta(1+r)\left[N_1^e \cdot u'\big(\underbrace{1-\tau_1^e+(1+r)a_0}_{c_1^e \text{ if employed}}\big) + (1-N_1^e)\cdot u'\big(\underbrace{b_1^e-\tau_1^e+(1+r)a_0}_{c_1^e \text{ if unemployed}}\big)\right] \tag{5}$$

As is standard in models at the zero liquidity limit (e.g., Werning 2015), we assume that at least one Euler equation holds with equality. As we show in appendix A.1, this will be the employed workers' Euler equation, because they have a stronger incentive to save in period 0. Then, (5) implies that the consumption of employed workers in period 0, $c_0(E)$, is increasing in the expectations for employment in period 1, N_1^e.

Policy. The government runs a balanced budget

$$\tau_t = (1-N_t)b_t \tag{6}$$

We specify the different implementations of unemployment benefits b_t below in the context of the business cycle stabilization experiment.

We assume that monetary policy target and implements a level for nominal GDP:

$$P_t C_t = M_t \tag{7}$$

where P_t is the price level, C_t is aggregate consumption, and M_t is the nominal GDP target. We assume that prices are fully rigid and normalize the price level to one, $P_t \equiv 1$. The monetary authority sets the level of nominal GDP at time 1, M_1, and the real rate between periods 0 and 1, r. Let M_0 adjust to support the equilibrium given exogenous monetary policy (r, M_1).

In this simple model, we consider an exogenous shock to time-1 nominal GDP M_1. The combination of sticky prices and the cash-in-advance constraint (7) implies that these shocks also affect aggregate quantities. As a result, these assumptions allow us to consider demand shocks in this simple two-period model.

Equilibrium. Given initial assets a_{-1}, exogenous variables $\{b_t, r, M_1\}$, and beliefs $\{N_1^e, b_1^e, \tau_1^e\}$, a temporary equilibrium is a collection of prices $\{w_t\}$ and allocations $\{c_t^E, c_t^U, N_t, \tau_t, M_0\}$ such that the representative firms optimizes, households optimize, government budget is balanced, the cash in advance constraint is satisfied, goods market clears

$$Y_t = C_t = N_t c_t^E + (1 - N_t) c_t^U \tag{8}$$

and asset market clears

$$0 = A_t = N_t a_t^E + (1 - N_t) a_t^U \tag{9}$$

The formal derivation of the model solution is relegated to appendix A.1. In the zero liquidity limit, the model is purely forward-looking. So the time-1 equilibrium is independent of time-0 outcomes, including the expectations that households hold in period 0. However, since employed workers are on the Euler equation, their expectations are relevant for equilibrium in period 0. As such, the model isolates the effect of imperfect anticipation of benefits in general equilibrium (GE).

2.2 Beliefs

For the purposes of this section, we impose a simple model of beliefs, based on the idea of belief distortion about future deviations from steady state. Formally, we assume that, for a given variable x_1, households' beliefs are given by

$$dx_1^e = \lambda dx_1 \tag{10}$$

where λ is a cognitive bias. Note that $\lambda = 1$ corresponds to full-information and rational expectations (FIRE). If $\lambda < 1$, then beliefs under-react relative to FIRE. If $\lambda > 1$, then beliefs over-react relative to FIRE. As we discuss next, whether beliefs under-react or over-react with respect to the FIRE benchmark is essential in addressing our questions.

Do beliefs under-react or over-react to innovations in fundamentals? This question has been the focus of an extensive empirical literature looking at survey evidence, but a consensus has not been reached. For instance, Coibion and Gorodnichenko (2012, 2015) find evidence of belief under-reaction. This finding is consistent with models of rational inattention or information rigidities, as in Sims (2003), Woodford (2001), Carroll (2003), Mankiw and Reis (2002), or Gabaix (2020). Instead, Bordalo, Gennaioli, Ma and Shleifer (2020) find evidence of belief over-reaction, which is consistent with models of diagnostic expectations and overextrapolation as in Bordalo, Gennaioli and Shleifer (2018). More recently, Angeletos, Huo and Sastry (2021) find evidence of initial under-reaction and a pattern of delayed over-reaction. Given the central importance of expectations in our analysis, in Section 3, we present a framework that can accommodate arbitrary deviations from FIRE and, in Section 4, we use this framework to directly match the empirical behavior of beliefs in surveys of expectations.

2.3 Macroeconomic stabilization

We demonstrate that deviations from FIRE affect the power of unemployment benefit extensions to stabilize aggregate demand. To this end, we induce a recession at time $t = 1$ and characterize the first-order change in equilibrium at time $t = 0$ from anticipating the recession.

The recession originates in a decrease in time-1 nominal GDP, $dM_1 < 0$, that translates one-to-one into lower employment, $dN_1 = dM_1$. In response, employed households will try to save more in period 0 according to the Euler equation (5). Since they cannot save in equilibrium, their time-0 consumption has to fall to dissuade them from saving. Thus a recession arises endogenously in period 0. The recession's severity depends on the strength of households' precautionary saving motive which depends on expected unemployment benefits. We consider two implementations of countercyclical UI benefits b_1. In both cases, the other policy instruments $\{r, b_0\}$ remain constant.

1. **Instrument rule.** The government announces that unemployment benefits are indexed to the unemployment rate according to a rule

$$db_1^{\text{rule}} = -\zeta_b \cdot dN_1 \tag{11}$$

with semi-elasticity $\zeta_b > 0$. We assume that households understand the policy announcement and has first-order knowledge of the rule that determines unemployment benefits. It follows that their expectations of unemployment benefits are given by:

$$db_1^e = -\zeta_b dN_1^e \tag{12}$$

2. **Instrument announcement.** We also consider a counterfactual scenario in which the government announces the change in unemployment benefits, db_1^{ann}, directly. We assume that

households understand the policy announcement and update their expectations accordingly

$$db_1^e = db_1^{\text{ann}} \tag{13}$$

To make the two policies comparable, we assume that they implement the same transfers, i.e. $db_1^{\text{ann}} = db_1^{\text{rule}}$. This implies that the time-1 equilibrium $\{dN_1, d\tau_1, dc_1(E), dc_1(U)\}$ is the same under both policies. By extension, beliefs $dN_1^e = \lambda dN_1$ and $d\tau_1^e = \lambda\tau_1$ are also the same under the two policies. However, under a rule, the inability to predict the unemployment rate also translates to an inability of predicting the policy stance, and so $db_1^{e,rule} = \lambda db_1^{e,\text{ann}}$. Proposition 1, the main result of this section, shows the implications for the endogenous recession in period 0.

Proposition 1. *Consider a shock dM_1 to nominal GDP in period 1. Let's assume that the government responds by announcing an unemployment benefit extension db_1 in one of two ways: either according to the rule* (11) *or a direct announcement. The first-order impact of the shock on time-0 output under these two regimes are*

$$dY_0^{rule} - dY_0^{ann} = \mathcal{M} \cdot M_b \cdot \zeta_b \cdot (1-\lambda) dU_1 \tag{14}$$

*where the first term is a standard **Keynesian multiplier***

$$\mathcal{M} = \frac{1}{1 - \frac{\partial C_0}{\partial Y_0}} = \frac{1}{b_0} \tag{15}$$

*the second term is the **marginal propensity to consume** out of anticipated unemployment benefits*

$$M_b = \frac{\partial c_0(E)}{\partial b_1} = \frac{\beta(1+r)(1-N_1^e) \cdot u''(b_1^e - \tau_1^e)}{u''(1-\tau_0)} \tag{16}$$

*and the third term $(1-\lambda)$ is a **cognitive bias**.*

Equation (14) shows that implementing the same UI extension, db_1, as a rule or a direct announcement can affect the severity of the recession in period 0. Under FIRE ($\lambda = 1$), households forecast the unemployment rate perfectly and infer the correct level of benefits, leading to $dY_0^{\text{rule}} - dY_0^{\text{ann}} = 0$. So, deviating from FIRE is a necessary condition for the implementation of UI extension to make a difference.

If $\lambda \neq 1$, households erroneously forecast the increase in the unemployment rate at time 1. Under the rules-based policy, this also means that they make mistakes in forecasting the increase in generosity of unemployment benefits. In other words, their misperception of tomorrow's unemployment rate also translates into a misperception of the future policy stance. Their forecast error is given by their cognitive bias $1-\lambda$ multiplied by the change in benefits $\zeta_b dM_1$. Instead, under the instrument-announcement policy, the household understands the policy announcement and so their expectations of unemployment benefits will always be correct.

So, if household beliefs under-react relative to FIRE, $\lambda < 1$, forecast mistakes make the rules-based policy less effective than the instrument-announcement policy. The efficiency loss is characterized by two sufficient statistics. First, the MPC out of anticipated benefits, M_b. This captures the partial equilibrium effect of underestimating UI benefits on aggregate demand. Second, the multiplier, $\mathcal{M}$, which captures the general equilibrium feedback from a contemporaneous change in aggregate demand. Instead, if household beliefs over-react relative to FIRE, $\lambda > 1$, forecast mistakes make the rules-based policy more effective than the instrument-announcement policy.

3 A framework for dynamic models with imperfect expectations

Next we lay out a framework of dynamic decision making with imperfect expectations. Our framework has two components. First, a model of how actions evolve given any expectations. Second, a model of how expectations are formed from observations. In appendix B, we map many popular models of bounded rationality and information frictions into our framework.

3.1 Dynamic decisions with general deviations from FIRE

Consider a forward-looking agent who chooses an output Y_t over periods $t = 0, 1 \ldots, T-1$. Let the vector $\mathbf{Y} \in \mathbb{R}^T$ denote the path of the output. For ease of exposition, let every object (parameters, initial-, and terminal conditions) that matters for the decision be fixed and known to the agent except the path of a single univariate input $\mathbf{X} \in \mathbb{R}^T$. The extension to multiple time-varying inputs is straightforward.

Auclert, Bardóczy, Rognlie and Straub (2021) cast such dynamic decision problems as a mapping between sequences

$$\mathbf{Y} = f(\mathbf{X}) \tag{17}$$

Their SSJ method computes the Jacobian $\mathcal{J} \in \mathbb{R}^{T\times T}$ then computes impulse responses to any shock $d\mathbf{X}$ via matrix multiplication, $d\mathbf{Y} = \mathcal{J} d\mathbf{X}$.[4] The representation (17) is valid under two assumptions. First, certainty equivalence with respect to $\mathbf{X}$. When the agent chooses Y_t, she considers only her time-t expectations $\mathbf{X}^{e,t} \in \mathbb{R}^T$, not the entire distribution of $\mathbf{X}$. Second, perfect foresight (FIRE) with respect to $\mathbf{X}$. The agent's expectations are correct, $\mathbf{X}^{e,t} = \mathbf{X}$.

We are interested in a generalization of this setup which relaxes the assumption of FIRE. We retain certainty equivalence, so only the mean expectation matters. However, expectations may not be correct and may evolve over time. In period 0, the agent expects a path $\mathbf{X}^{e,0}$; in period 1, she expects a path $\mathbf{X}^{e,1}$; and so on. Each vector $\mathbf{X}^{e,\tau} = \begin{bmatrix} X_0^{e,\tau} & X_1^{e,\tau} & \ldots & X_T^{e,\tau} \end{bmatrix}'$ captures the beliefs that the agent holds at time τ about the variable X at every date. We assume that the agent observes current and past realizations (or, alternatively, all current realizations and the state variables for their individual decision making), and also assume that the agent does not foresee their future forecast errors (i.e., they are naive). So $X_t^{e,\tau} = X_t$ for all $t \leq \tau$. This ensures that the agent does not violate any constraints. In sum, relaxing FIRE implies that we have to keep track of the entire

[4]The Jacobian is computed at a baseline path $\bar{\mathbf{X}}$, typically a constant path corresponding to the steady state $\bar{\mathbf{Y}} = f(\bar{\mathbf{X}})$. So the shock $d\mathbf{X} = \mathbf{X} - \bar{\mathbf{X}}$ and the impulse response $d\mathbf{Y} = \mathbf{Y} - \bar{\mathbf{Y}}$ are both deviations from the baseline path.

history of expectations, $\mathbf{X}^{e,t}$ for all t. Formally,

$$\mathbf{Y} = g\left(\mathbf{X}, \{\mathbf{X}^{e,t}\}_t\right) \tag{18}$$

Conceptually it is clear that if we could compute all the Jacobians of $g(\bullet)$, we could compute linearized impulse responses. But the domain of $g(\bullet)$ is $\mathbb{R}^{T+T\times T}$, a much larger space than the domain of $f(\bullet)$ which is just $\mathbb{R}^T$. Is this approach viable in practice? Propositions 2 and 3 show that it is. The key idea is to manipulate the FIRE Jacobian $\mathcal{J}$ to capture the responses to forecast errors. This insight appears in Auclert, Rognlie and Straub (2020), who implemented specific deviations from FIRE via Jacobian manipulation.[5] Propositions 2 and 3 do the same for general deviations from FIRE, using the familiar concepts of forecast errors and forecast revisions.

Proposition 2 handles the special case of non-rational but time-invariant expectations $d\mathbf{X}^e \neq d\mathbf{X}$. An example of this is level-k thinking. The total response $d\mathbf{Y}$ is the sum of two effects. First, the response to the expected part of the shock. Second, the responses to the forecast errors that the agent observes along the way. The key new object is the forecast-error Jacobian, $\mathcal{E}$, that captures the second effect. Column s of $\mathcal{E}$ can interpreted as the impulse response to the forecast error in dX_s which the agent learns in period s. Constructing $\mathcal{E}$ is straightforward. It is a lower diagonal matrix whose columns are shifted versions of the first column of $\mathcal{J}$. The intuition is that observing a forecast error in period t is equivalent to observing an unexpected shock in period 0. The formal proof is in appendix B.1.

Proposition 2. *Assuming constant beliefs $X^{e,t}_{t+h} = X^{e,0}_{t+h}$ for all $t, h > 0$, the linearized impulse response $d\boldsymbol{Y}$ to an arbitrary shock $d\boldsymbol{X}$ is given by*

$$d\boldsymbol{Y} = \mathcal{J}\underbrace{d\boldsymbol{X}^{e,0}}_{\text{forecast}} + \mathcal{E}\underbrace{\left(d\boldsymbol{X} - d\boldsymbol{X}^{e,0}\right)}_{\text{forecast error}} \tag{19}$$

*where the **forecast-error Jacobian** $\mathcal{E}$ is given by*

$$\mathcal{E} = \begin{bmatrix} \mathcal{J}_{0,0} & 0 & \dots & 0 \\ \mathcal{J}_{1,0} & \mathcal{J}_{0,0} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \mathcal{J}_{t,0} & \mathcal{J}_{t-1,0} & \dots & \mathcal{J}_{0,0} \end{bmatrix} \tag{20}$$

Proposition 3 handles the general case of time-variant expectations. The new element is that observing the forecast error $dX_t - dX^{e,t-1}_t$ may cause the agent to update her expectations for all future periods. Capturing this effect is most straightforward if we work with forecast revisions $d\boldsymbol{X}^{e,h} - d\boldsymbol{X}^{e,h-1}$ instead of forecast errors $d\boldsymbol{X} - d\boldsymbol{X}^{e,h}$. The Jacobians that act on forecast revision vectors are simply shifted versions of the FIRE Jacobian $\mathcal{J}$. The intuition is that a forecast revision

[5] Appendix D.3 of Auclert, Rognlie and Straub (2020) provides recipes to implement sticky expectations, cognitive discounting, and dispersed information.

for periods $t, \ldots, T-1$ is equivalent to observing an unanticipated shock for periods $0, \ldots, T-t$. The formal proof is in appendix B.2.

Proposition 3. *Assuming time-variant beliefs $\boldsymbol{X}^{e,t}$, the linearized impulse response $d\boldsymbol{Y}$ to an arbitrary shock $d\boldsymbol{X}$ is given by*

$$d\boldsymbol{Y} = \mathcal{J} \underbrace{d\boldsymbol{X}^{e,0}}_{\text{initial forecast}} + \sum_{h \geq 1} \mathcal{R}_h \underbrace{\left(d\boldsymbol{X}^{e,h} - d\boldsymbol{X}^{e,h-1} \right)}_{\text{forecast revision}} \tag{21}$$

*where the **forecast-revision Jacobian** $\mathcal{R}_h$ for any $h > 1$ is given by*

$$\mathcal{R}_h = \begin{bmatrix} 0 & \mathbf{0}'_h \\ \mathbf{0}_h & \mathcal{J} \end{bmatrix}$$

Application to heterogeneous-agent models. Propositions 2 and 3 apply to heterogeneous-agent models in which $Y_t = \int y_t dD_t$ is an aggregate of individual decisions y_t for some non-trivial, time-varying distribution D_t. However, we need to impose restrictions on belief heterogeneity. In the exposition above, we assume that everyone has the same beliefs. More generally, this framework can be directly used even if expectations are heterogeneous as long as they are uncorrelated with other idiosyncratic characteristics in the cross-section. For this purpose, we redefine $X^{e,t}$ as the cross-sectional average expectation.

It is also possible to use this framework to allow for meaningful belief disagreement as long as beliefs are with permanent individual characteristics. In this case, one has to set up a heterogeneous-agent block for each permanent type, and apply the propositions type by type. Guerreiro (2022) follows this approach in his study of disagreements over the business cycle.

3.2 A flexible model of expectations

Propositions 2 and 3 enable us to compute linearized impulse responses to any shock $d\mathbf{X}$ given the path of expectations $\{d\mathbf{X}^{e,t}\}_t$ conditional on the same shock.[6] In some cases, the response of expectations may be estimated directly. We'll do so in section 4.6 with respect to unemployment. Another route is to impose a model of expectation formation. In this subsection, we present a tractable yet flexible specification that nests many popular models including: (1) sticky expectations (Mankiw and Reis, 2002, Carroll, Crawley, Slacalek, Tokuoka and White, 2018), (2) noisy-information and rational expectations (Angeletos and Huo, 2021), (3) cognitive discounting (Gabaix, 2020), (4), sparsity (Gabaix, 2014, 2016, Guerreiro, 2022), (5) shallow reasoning (Angeletos and Sastry, 2021), (6) finite planning horizons (Woodford, 2018), (7) adaptive expectations (Cagan, 1956, Friedman, 1957), (8) diagnostic expectations (Bordalo, Gennaioli and Shleifer, 2018, Bianchi, Ilut and Saijo, 2021), (9) noisy-information diagnostic expectations (Bordalo, Gennaioli, Ma and Shleifer, 2020), among others. Appendix B.3 discusses how to map each of these models into our framework.

[6] Recall that deviations $d\mathbf{X}$ and $\{d\mathbf{X}^{e,t}\}_t$ are all relative to the paths around which one wishes to compute the Jacobian.

Propositions 2 and 3 deal with linear mappings in sequence space. So it's natural for us to model expectations in the same way. The most general linear sequence-space model of expectations we can write down—given a single time-variant input $d\mathbf{X}$—is a sequence of matrices $\Lambda_t \in \mathbb{R}^{T\times T}$ that map realized outcomes $d\mathbf{X} \in \mathbb{R}^T$ into time-t expectations $d\mathbf{X}^{e,t} \in \mathbb{R}^T$ according to

$$d\mathbf{X}^{e,t} = \Lambda_t d\mathbf{X} \tag{22}$$

We maintain the assumption that expectations of current and past realizations of $d\mathbf{X}$ are correct. This implies that the upper-left block of Λ_t is the identity matrix

$$\Lambda_t = \begin{bmatrix} \mathbf{I}_{t\times t} & \cdots \\ \vdots & \ddots \end{bmatrix} \tag{23}$$

Equation (22) looks simple but it can capture rich theories of expectation formation. It can account for an understanding of the data generating process as well as for updating of priors in light of new observations. In our expository environment, $d\mathbf{X}$ is the only input the agent has to form expectations about. So it's not restrictive to assume that $d\mathbf{X}^{e,t}$ depends only on the realized path of $d\mathbf{X}$ itself. In richer environments with multiple inputs, one may introduce additional linear terms, allowing the agent to think about cross-equation restrictions directly. For example, expectations of unemployment benefits and income taxes may be related via an understanding of the government budget constraint.

Corollaries 1 and 2 substitute (22) into propositions 2 and 3. The bracketed terms can be interpreted as the Jacobians of the non-FIRE decision problem represented by the function $g\left(\mathbf{X}, \{\mathbf{X}^{e,t}\}_t\right)$. These non-FIRE Jacobians account for the direct effect of $d\mathbf{X}$ on $d\mathbf{Y}$ as well its indirect effect through $\{\mathbf{X}^{e,t}\}_t$. Crucially, they're still $T \times T$ matrices just like the FIRE Jacobians of $f(\mathbf{X})$. So, the rest of the SSJ machinery of Auclert, Bardóczy, Rognlie and Straub (2021) applies without further modifications. In sum, we're now equipped to solve dynamic general equilibrium models with (or without) rich heterogeneity under general deviations from FIRE.

Corollary 1. *Consider the setup of proposition 2 with FIRE Jacobian $\mathcal{J}$, and forecast-error Jacobian $\mathcal{E}$. Let the constant expectations be $d\boldsymbol{X}^e = \Lambda d\boldsymbol{X}$, according to* (22)*. The linearized impulse response $d\boldsymbol{Y}$ to an arbitrary shock $d\boldsymbol{X}$ is*

$$d\boldsymbol{Y} = \Big[\left(\mathcal{J} - \mathcal{E}\right)\Lambda + \mathcal{E}\Big] d\boldsymbol{X} \tag{24}$$

Corollary 2. *Consider the setup of proposition 3 with FIRE Jacobian $\mathcal{J}$, and forecast-revision Jacobians $\mathcal{R}^\tau$. Let expectations be $d\boldsymbol{X}^{e,t} = \Lambda_t d\boldsymbol{X}$, according to* (22)*. The linearized impulse response $d\boldsymbol{Y}$ to an arbitrary shock $d\boldsymbol{X}$ is*

$$d\boldsymbol{Y} = \left[\mathcal{J}\Lambda_0 + \sum_{\tau\geq 1} \mathcal{R}_\tau \left(\Lambda_\tau - \Lambda_{\tau-1}\right)\right] d\boldsymbol{X} \tag{25}$$

Given a model for beliefs, the matrices given in equations (24) or (25) fully summarize the response of the heterogeneous agent block to the path $d\mathbf{X}$, taking into account forecast errors and revisions. These matrices are easy and fast to compute after obtaining the FIRE Jacobians $\mathcal{J}$.

4 HANK model with imperfect expectations

Next we present a full-fledged dynamic general equilibrium model that's suitable for a quantitative evaluation of unemployment benefit extensions with imperfect expectations. Proposition 1 highlights several features that we view as crucial for this exercise:

$$dY_0^{\text{rule}} - dY_0^{\text{ann}} = -\underbrace{\mathcal{M}}_{\text{multiplier}} \cdot \underbrace{M_b}_{\text{aggregate demand response to UI}} \cdot \underbrace{\zeta_b(1-\lambda)dU_1}_{\text{forecast error of UI extension}} \tag{26}$$

First, we would like anticipated unemployment spells and unemployment benefits to have a reasonable impact on consumption. This requires that households have precautionary saving motive with respect to unemployment. As in our illustrative model, we assume that households are prudent and markets are incomplete. Quantitatively, it matters that households are heterogeneous in the degree of self-insurance against unemployment risk. Some households have ample savings to ride out a typical unemployment spell, while others are dependent on unemployment benefits. Second, we need a model with a reasonable feedback from aggregate demand to equilibrium output and employment. That is, it matters how aggregate demand translates into a distribution of income (from labor, capital, and transfers) over time, the expectations of these incomes, and the responses of consumption and investment.

In light of these considerations, we propose a New Keynesian model with heterogeneous households, search and matching unemployment, sticky prices and wages, investment adjustment costs, and smooth fiscal policy (gradual tax adjustments, long-term bonds). We build on models of automatic stabilizers (McKay and Reis, 2016, Kekre, 2021), and estimated medium-scale New Keynesian models (Christiano, Eichenbaum and Trabandt, 2016, Auclert, Rognlie and Straub, 2020). Appendix C contains detailed derivations of the equilibrium conditions.

4.1 Households

The household block is a standard incomplete markets model. There is a unit mass of ex-ante identical households. In any given period, households are heterogeneous with respect to employment status $e_{it} \in \{E, U, N\}$, productivity $z_{it} \in \mathcal{G}_z$, and liquid assets $a_{it-1} \geq \underline{a}$. The model frequency is quarterly and timing is as follows.

1. **Productivity shock.** Households draw a new productivity z_{it} from a finite set $\mathcal{G}_z$. Productivity follows a discrete Markov process with fixed transition matrix Π_z.

2. **Labor market transitions.** First, employed workers lose their job with probability s_t. Second, unemployed workers (including those who separated in this quarter) find jobs with the endogenous probability f_t. Third, newly unemployed workers qualify for unemployment

benefits with probability π^{get}, while other households on UI lose eligibility with probability π_t^{lose}. The probability of losing UI eligibility maps directly to the expected duration of benefits $1/\pi_t^{lose}$ and is the key policy variable. The combined transition matrix for labor market status e_{it} is

$$\begin{array}{cc} & \begin{array}{ccc} E_t & U_t & N_t \end{array} \\ \begin{array}{c} E_{t-1} \\ U_{t-1} \\ N_{t-1} \end{array} & \begin{pmatrix} 1 - s_t(1-f_t) & \pi^{get} s_t(1-f_t) & (1-\pi^{get}) s_t (1-f_t) \\ f_t & (1-\pi_t^{lose})(1-f_t) & \pi_t^{lose}(1-f_t) \\ f_t & 0 & 1-f_t \end{pmatrix} \end{array} \tag{27}$$

3. **Consumption-saving decision.** Households choose consumption c_{it} and liquid assets a_{it} to maximize their expected lifetime utility subject to a budget constraint and a borrowing constraint.

The Bellman equation at the consumption-saving stage is

$$\begin{aligned} V_t(e_{it}, z_{it}, a_{it-1}) = & \max_{c_{it}, a_{it}} u(c_{it}) + \beta_t E_t[V_{t+1}(e_{it+1}, z_{it+1}, a_{it})] \\ \text{s.t. } c_{it} + a_{it} = & (1 + r_{t-1}^a) a_{it-1} + (1 - \tau_t) \Big[y_t(e_{it}, z_{it}) + d_t^{FI}(a_{it-1}) \Big] + T_t \\ y_t(e_{it}, z_{it}) = & w_t z_{it} \mathbb{1}\{e_{it} = E\} + b_t z_{it} \mathbb{1}\{e_{it} = U\} \\ a_{it} \geq & \underline{a} \end{aligned} \tag{28}$$

Households in state E are employed and earn labor income $w_t z_{it}$. Households in state U are unemployed and receive UI benefits $b_t z_{it}$. Households in state N are unemployed and have exhausted their UI benefits. The lump-sum transfer T_t ensures that every household can maintain positive consumption. Liquid assets are held as short-term deposits that earn riskless return r_{t-1}^a. Households also receive transfers $d_t^{FI}(a_{it-1})$ from a financial intermediary. These are indexed to liquid wealth, but are lump-sum in the sense that households don't internalize that accumulating more wealth will increase this transfer.[7]

4.2 Financial intermediary

All assets in the economy are held by a representative financial intermediary. The assets are three: shares in firm equity v_t, long-term nominal government bonds B_t, and short-term nominal reserves M_t. The liabilities of the financial intermediary are net worth N_t^{FI} and short-term deposits

[7] This is a simple way to introduce illiquid assets that has three important benefits. First, the model is as easy to solve as any 1-asset model. Second, similarly to full-fledged two-asset models, it can reconcile high average MPC with a realistic amount of assets (including capital). Third, together with the setup of the financial intermediary, the model can match moderate MPC out of asset price fluctuations, which is a challenge for standard two-asset models even with large portfolio adjustment costs and imperfect expectations.

A_t from households. Thus the balance sheet, in date-t real terms, is

$$p_t v_t + q_t^B \frac{B_t^N}{P_t} + \frac{M_t}{P_t} = N_t^{FI} + A_t \tag{29}$$

where P_t is the price level, p_t is the equity price, q_t^B is price of long nominal bonds, and the price of reserves is 1. Going forward, let $\pi_t = P_t/P_{t-1} - 1$ denote the inflation rate.

The nominal return on these assets are the following. One share of equity purchased in period $t-1$ yields dividend stream $\{P_{t+s}d_{t+s}\}$ for all $s \geq 0$. One government bond purchased in period $t-1$ pays a coupon δ_B^s in period $t+s$ for all $s \geq 0$. One unit of reserves purchased in period $t-1$ pays $(1+i_{t-1})$ in period t. Finally, the intermediary pays out d_t^{FI} as dividend to households in period t. This implies that net worth is

$$N_t^{FI} = \underbrace{(d_t + p_t)v_{t-1}}_{\text{gross return on equity}} + \underbrace{\frac{1+\delta_B q_t^B}{1+\pi_t}\frac{B_{t-1}}{P_{t-1}} + \frac{1+i_{t-1}}{1+\pi_t}\frac{M_{t-1}}{P_{t-1}}}_{\text{gross return on nominal assets}} - \underbrace{(1+r_{t-1}^a)A_{t-1} - d_t^{FI}}_{\text{pay to households}} \tag{30}$$

Payouts to households follow an ad hoc rule

$$\log\left(\frac{d_t^{FI}}{d_{ss}^{FI}}\right) = \phi_N \log\left(\frac{N_{t-1}^{FI}}{N_{ss}^{FI}}\right) \tag{31}$$

By choosing a low ϕ_N, we can smooth out the financial income of households relative to fluctuations in the underlying asset prices. This prevents counterfactually large consumption responses out of asset price fluctuations.

The financial intermediary chooses v_t, B_t^N, A_t, and M_t to maximize its expected return on net worth, $\mathbb{E}_t[N_{t+1}^{FI}/N_t^{FI}]$, subject to the constraints (30) and (31). This yields the no arbitrage conditions

$$1 + r_t^a = \mathbb{E}_t\left[\frac{d_{t+1}+p_{t+1}}{p_t}\right] = \mathbb{E}_t\left[\frac{1+\delta_B q_{t+1}^B}{q_t^B(1+\pi_{t+1})}\right] = \mathbb{E}_t\left[\frac{1+i_t}{1+\pi_{t+1}}\right] \equiv 1 + r_t \tag{32}$$

where we defined r_t as the economy-wide ex-ante real interest rate.

4.3 Firms

Our specification of firms is standard. We consider three sectors: retailers (nominal rigidities), capital producer (investment adjustment cost), and labor agency (search and matching frictions). These sectors are connected by competitive markets, so one could model them as one type of firm that makes the same decisions subject to the same constraints.

Retailers. There is unit mass of retailers indexed by j who engage in monopolistic competition. They produce differentiated goods using a Cobb-Douglas production function with the same productivity $y_{jt} = \Theta_t k_{jt}^\alpha n_{jt}^{1-\alpha}$. Firms hire capital k_{jt} and labor n_{jt} on spot markets at prices r_t^k and h_t

and pay a fixed cost Ξ. They also set the price of their product, p_{jt}, subject to a demand curve with constant elasticity ϵ and a quadratic price adjustment cost à la Rotemberg (1982). We allow for price indexation, so the adjustment cost is paid on price changes relative to a fraction ι_p of last period's price change. The firms' objective is to maximize the present value of their future profits. The Bellman equation is

$$
\begin{aligned}
J_t^R(p_{jt-1}, p_{jt-2}) = \max_{k_{jt}, n_{jt}, y_{jt}, p_{jt}} & \left\{ \frac{p_{jt}}{P_t} y_{jt} - h_t n_{jt} - r_t^K k_{jt} - \Psi_{jt}^p - \Xi + E_t \left[\frac{J_{t+1}^R(p_{jt}, p_{jt-1})}{1+r_t} \right] \right\} \\
\text{s.t. } y_{jt} &= \Theta_t k_{jt}^{\alpha} n_{jt}^{1-\alpha} \\
y_{jt} &= \left(\frac{p_{jt}}{P_t} \right)^{-\epsilon} Y_t \\
\Psi_{jt}^p &= \frac{\psi_p}{2} \left[\log\left(\frac{p_{jt}}{p_{jt-1}} \right) - \iota_p \log\left(\frac{p_{jt-1}}{p_{jt-2}} \right) \right]^2 Y_t
\end{aligned}
$$

In a symmetric equilibrium, all firms choose the same level of output, capital, and labor. So, they have the same marginal cost:

$$
mc_t = \frac{1}{\Theta_t} \left(\frac{r_t^K}{\alpha} \right)^{\alpha} \left(\frac{h_t}{1-\alpha} \right)^{1-\alpha} \tag{33}
$$

and set the same prices according to the Phillips curve

$$
\pi_t - \iota_p \pi_{t-1} = \frac{\psi_p}{\epsilon} \left(mc_t - \frac{\epsilon - 1}{\epsilon} \right) + \frac{1}{1+r_t} E_t \left[\frac{Y_{t+1}}{Y_t} \left(\pi_{t+1} - \iota_p \pi_t \right) \right] \tag{34}
$$

Capital producer. A representative firm owns the capital stock and rents it to retailers at rate r_t^K. It's Bellman equation is

$$
\begin{aligned}
J_t^K(K_{t-1}, I_{t-1}) = \max_{K_t, I_t} & \left\{ r_t^K K_{t-1} - I_t + E_t \left[\frac{J_{t+1}^K(K_t, I_t)}{1+r_t} \right] \right\} \\
\text{s.t. } K_t &= (1-\delta) K_{t-1} + \mu_t \left[1 - S\left(\frac{I_t}{I_{t-1}} \right) \right] I_t
\end{aligned} \tag{35}
$$

where $\delta \in (0, 1)$ is the depreciation rate, I_t is investment, μ_t is the marginal efficiency of investment as in Justiniano, Primiceri and Tambalotti (2010), and $S(\bullet)$ is a convex function that satisfies $S(1) = S'(1) = 0$.

Defining Tobin's Q as the marginal value of capital at the end of period t, investment dynamics is characterized by

$$
Q_t = \frac{r_{t+1}^K + E_t \left[Q_{t+1} (1-\delta) \right]}{1+r_t} \tag{36}
$$

$$
1 = Q_t \mu_t \left[1 - S\left(\frac{I_t}{I_{t-1}} \right) - \left(\frac{I_t}{I_{t-1}} \right) S'\left(\frac{I_t}{I_{t-1}} \right) \right] + E_t \left[\frac{\mu_{t+1} Q_{t+1}}{1+r_t} \left(\frac{I_{t+1}}{I_t} \right)^2 S'\left(\frac{I_{t+1}}{I_t} \right) \right] \tag{37}
$$

Labor agency. A representative firm hires workers on a frictional labor market and rents homogeneous labor services to retailers at rate h_t. The agency posts vacancies v_t, each of which is filled with probability q_t. Following Christiano, Eichenbaum and Trabandt (2016), we assume a two-tiered cost of hiring. The firm pays κ_v to create a vacancy and then κ_h for each vacancy it fills.[8] Incumbent workers separate with probability s_t. The Bellman equation is

$$J_t^L(N_{t-1}) = \max_{N_t, v_t} \left\{ (h_t - w_t)N_t - (\kappa_v + \kappa_h q_t)v_t + E_t \left[\frac{J_{t+1}^L(N_t)}{1+r_t} \right] \right\} \tag{38}$$
$$\text{s.t. } N_t = (1-s_t)N_{t-1} + q_t v_t$$

Optimization yields a standard job creation curve, equating the cost and benefit of hiring the marginal worker

$$\frac{\kappa_v}{q_t} + \kappa_h = h_t - w_t + E_t \left[\frac{1 - s_{t+1}}{1+r_t} \left(\frac{\kappa}{q_{t+1}} + \kappa_h \right) \right] \tag{39}$$

4.4 Government policy

The fiscal authority issues long-term nominal bonds, collects income taxes, and provides unemployment benefits. Let U_t denote the mass of workers eligible for unemployment benefits. The government budget constraint is

$$G_t + T_t + (1-\tau_t)b_t U_t + \frac{(1+\delta_B q_t^B)}{1+\pi_t} \frac{B_{t-1}}{P_{t-1}} = \tau_t(w_t N_t + d_t^{FI}) + q_t^B \frac{B_t}{P_t} \tag{40}$$

Spending G_t and lump-sum transfers T_t are exogenous. The income tax rate τ_t is chosen according to a rule that can prevent large swings in the tax rate, while ensuring that real government debt is stationary

$$\tau_t - \tau_{ss} = \phi_B q_{ss}^B \left(\frac{B_{t-1}}{P_{t-1}} - \frac{B_{ss}}{P_{ss}} \right) \tag{41}$$

In the announcement-based policy, UI duration $1/\pi_t^{lose}$ is exogenous. In the rule-based policy, it is indexed to the end-of-period unemployment rate

$$\frac{1}{\pi_t^{lose}} - \frac{1}{\pi_{ss}^{lose}} = -\zeta_b (N_t - N_{ss}) \tag{42}$$

The monetary authority sets the short-term nominal interest rate according to

$$i_t = \rho_m i_{t-1} + (1-\rho_m)(i_{ss} + \phi_\pi \pi_t) + \epsilon_t^m \tag{43}$$

where ϵ_t^m is a monetary policy shock.

[8] The role of κ_h is similar to wage stickiness in models without search and matching. It dampens the procyclicality of marginal costs and hence profits. This is especially important in HANK models where strongly countercyclical profits can have large—and unrealistic—redistributive effects (Broer, Harbo Hansen, Krusell and Öberg, 2020).

4.5 Equilibrium

Wage setting. A risk-neutral labor union bargains with the labor agency on behalf of employed workers. The surplus of the union is

$$H_t = w_t - b_t + E_t\left[\frac{(1-s_{t+1})(1-f_{t+1})}{1+r_t}H_{t+1}\right] \tag{44}$$

and the surplus of the labor agency is

$$J_t = h_t - w_t - \Psi^w(w_t, w_{t-1}) + E_t\left[\frac{1-s_{t+1}}{1+r_t}J_{t+1}\right] \tag{45}$$

where $\Psi_w(\bullet)$ is a convex function that captures real wage rigidities

$$\Psi^w(w_t, w_{t-1}) = \frac{\psi_w}{2}\left(\frac{w_t}{w_{t-1}} - 1\right)^2 \tag{46}$$

The real wage w_t is then set to maximize $H_t^{\eta} J_t^{1-\eta}$.

Matching. New matches are formed on the labor market according to a Cobb-Douglas matching function

$$M(JS_t, v_t) = A_m (JS_t)^{\ell} v_t^{1-\ell} \tag{47}$$

where the mass of job seekers equals the mass of unemployed workers from last period plus the mass of newly separated workers

$$JS_t = 1 - N_{t-1} + s_t N_{t-1} \tag{48}$$

Let $\theta_t \equiv v_t / JS_t$ denote labor market tightness. Job finding and vacancy filling probabilities are

$$f_t = A_m \theta_t^{1-\ell} \qquad \text{and} \qquad q_t = \frac{f_t}{\theta_t}. \tag{49}$$

Market clearing. Factor market clearing requires

$$N_t = \int n_{jt} dj \tag{50}$$

$$K_{t-1} = \int k_{jt} dj \tag{51}$$

The notation reflects that capital is predetermined from the perspective of capital producers but not from the perspective of retailers. Aggregate dividends are given by

$$d_t = d_t^R + d_t^K + d_t^L = Y_t - w_t N_t - I_t - \Psi_t^p - (\kappa_v + \kappa_h q_t) v_t - \Xi \tag{52}$$

Firm equity is then priced according to (32). Asset market clearing corresponds to the balance sheet of the financial intermediary (29), imposing that the intermediary holds all shares $v_t = 1$, and nominal reserves are zero $M_t = 0$. Nominal reserves are in zero net supply, the purpose of including them is to deliver a Fisher equation in (32). Goods market clearing requires that the final good is used for household consumption, investment (including adjustment costs), government spending, price adjustment costs, hiring costs, and the fixed cost.

$$Y_t = C_t + I_t + G_t + \Psi_t^p + (\kappa_v + \kappa_h q_t) v_t + \Xi \tag{53}$$

Definition. Given initial conditions for the distribution of households D_{-1}, net worth N_{-1}^{FI}, government debt B_{-1}, price level $\{P_{-1}, P_{-2}\}$, investment I_{-1}, capital K_{-1}, real wage w_{-1}, and sequences of exogenous variables $\{\beta_t, \Theta_t, \mu_t, s_t, G_t, T_t, \epsilon_t^m\}$, competitive equilibrium is a sequence of prices $\{P_t, w_t, h_t, r_t^K, r_t, q_t^B, p_t, \pi_t, i_t, r_t^a\}$, aggregates $\{Y_t, N_t, K_t, I_t, Q_t, v_t, q_t, f_t, \theta_t, B_t, \tau_t, \pi_t^{lose}, d_t^{FI}, b_t,$ $d_t, N_t^{FI}, mc_t, U_t, H_t, J_t\}$, policy functions $\{a_t, c_t\}$, and distributions $\{D_t\}$ such that households optimize, financial intermediary optimizes, firms optimize, monetary and fiscal authorities follow their rules, markets clear, job-finding and vacancy-filling probabilities are consistent with the matching function, and the employment and UI eligibility rates are consistent with the distribution's law of motion.

4.6 Estimation

We estimate the model in two steps, similarly to Christiano, Eichenbaum and Evans (2005) and Auclert, Rognlie and Straub (2020). In the first step, we pin down all the parameters that affect the steady state. We fix some parameters to conventional values from the literature, and calibrate others internally to hit steady-state moments. In the second step, we estimate the remaining parameters by impulse response matching.

Calibration of steady state. Households have CRRA utility over consumption $u(c) = c^{1-\sigma^{-1}}/(1-\sigma^{-1})$ with an EIS of $\sigma = 0.5$. We set the borrowing limit to $\underline{a} = 0$. We assume that the annual economy-wide real interest rate is $r = 2\%$. The discount factor β is calibrated internally to deliver 20% average quarterly MPC out of a one-time lump-sum transfer. Our Markov process for labor productivity $(\mathcal{G}_z, \Pi_z)$ is the discrete-time equivalent of the process estimated by Kaplan, Moll and Violante (2018). To account for progressive taxation, we scale down the cross-sectional variance of log productivity by $(1 - 0.181)^2$, where 0.181 is the degree of progressivity in the log-linear retention function of Heathcote, Storesletten and Violante (2017). Mean productivity is normalized to 1.

For labor market transitions, we set the job-finding rate to $f = 0.6$ and calibrate the separation rate to deliver an unemployment rate of 4.5%. We assume that UI benefits replace 50% of the steady-state wage, and all unemployed workers qualify for benefits initially, $\pi^{get} = 1$. In steady state, unemployment benefits last on average for 2 quarters, $\pi^{lose} = 0.5$. We set the lump-sum transfer to $T = 0.01$, enough to ensure that borrowing-constrained households who have exhausted their UI benefits can consume a positive amount. We set the vacancy filling rate to $q = 0.7$ quarterly, and assume that κ_h accounts for 94% of total search cost, leaving 6% for vacancy posting cost per hire κ_v/q. We calibrate the bargaining power of the union η such that total search cost is 7% of the quarterly wage of an average worker.

We calibrate total factor productivity, Θ, to normalize output to $Y = 1$. We set government debt, B/P, to 46% of annual output, and choose the coupon, δ_B, to match the average duration of U.S. government debt of 5 years. Having realistic duration prevents counterfactually large exposure of government budget to fluctuations in short-term interest rates. This matters in non-Ricardian models. We set government spending, G, to 16% of output, which leads to a marginal tax rate of $\tau = 0.27$. We set depreciation rate to $\delta_K = 0.083/4$ quarterly and calibrate the capital share α to match a quarterly capital to output ratio of 8.92. This implies that the steady-state labor share is 62%. The fix cost Ξ is calibrated to make total wealth $p + q_b B/P$ equal to 382% of annual output.

One of the most important transition-specific parameters is ζ_b, the semi-elasticity of average unemployment duration with respect to the unemployment rate. We pin this down from a linear approximation of the Emergency Unemployment Compensation (EUC08) program. Our goal is to work with a policy rule that is in the right ballpark, not to provide a serious quantitative evaluation of EUC08 per se.[9]

The unemployment rate in 2007Q1 was 4.6%, close to the steady state value of 4.5% in our model. Unemployment rate peaked at 10.1% in 2009Q4. During the same time, unemployment benefit duration was raised from 26 weeks to 99 weeks (in states with unemployment rate above 8.5%). So, our back of the envelope calculation for the semi-elasticity is $\zeta_b = (99-26)/13/(0.101 - 0.046) \approx 102$. That is, a one percentage point increase in the unemployment rate triggers 1.02 quarter increase in average UI duration.

Estimation: IRF Matching. As in Christiano, Eichenbaum and Evans (2005), we estimate the model by matching the impulse response functions obtained in our model to their empirical counterparts obtained with a standard business-cycle shock. We generate the empirical impulse responses by following the empirical strategy in Angeletos, Huo and Sastry (2021). That is, we estimate the regression

$$z_t = \alpha + \sum_{p=1}^{P} \gamma_p z_{t-p}^{IV} + \sum_{k=0}^{K} \beta_k \epsilon_{t-k} + u_t \tag{54}$$

where z_t is an outcome of interest (e.g. unemployment rate), ϵ_t is an identified shock, and z_{t-p}^{IV} are lagged values of z_t instrumented by lagged values of ϵ_t. Our identified shock is the main business cycle (MBC) shock from Angeletos, Collard and Dellas (2020). This shock is constructed to account for most of the business cycle fluctuations in unemployment rate. We generate impulse response

[9]EUC08 features nonlinearities and a staggered rollout which we ignore. Kekre (2021) for example takes these features into account but assumes perfect foresight with respect to the announced policy.

functions not only for outcomes, but also generate the impulse response functions of expectation for the relevant variables at several horizons.

To perturb the economy from its steady-state level, we consider a single shock to the marginal efficiency of investment (MEI) which follows an AR(1) process

$$\mu_t = \rho_\mu \mu_{t-1}.$$

The process for this shock has two free parameters: the persistence ρ_μ and the initial level of the shock μ_0. Angeletos, Collard and Dellas (2020) show that the MBC and MEI shocks are closely related. Using our model, we solve the impulse responses of variables in our model to this shock for a given set of parameters. In doing so, we attribute the estimated impulse responses for expectations directly into our household block using equation (21). We focus on the implications of imperfect expectations for aggregate demand and so assume that all other economic agents have full information and rational expectations.[10]

We recover the implied impulse response functions IRF(Ω). Ω denotes the set of that we estimate which can be seen in Table 2. We choose values for these parameters so as to minimize the distance between our model's implied impulse response and those estimated in the data:

$$\hat{\Omega} = \arg\min_{\Omega} \left(\mathrm{IRF}(\Omega) - \widehat{\mathrm{IRF}}\right)' \Sigma^{-1} \left(\mathrm{IRF}(\Omega) - \widehat{\mathrm{IRF}}\right),$$

where $\widehat{\mathrm{IRF}}$ denotes the estimated impulse response function. In our estimation, we include the impulse response functions for the unemployment rate, consumption, inflation rate, and the nominal interest rate.

Expectations: Data limitations and solution. In practice, the exercise described in the previous section cannot be fully implemented due to the unavailability of all necessary expectations data. In this dimension, we confront two limitations: (1) we may not have survey data on expectations to all relevant variables and (2) even for the variables for which we do have survey data for, we only observe expectations for a finite number of future horizons, and not the infinite number of horizons which would be required to solve the model.

We use data for the Survey of Professional Forecasters (SPF). From this dataset, we use data for one to four quarters-ahead unemployment rate forecasts.[11] However, people must still form expectations about the real-interest rate, tax rates, job-finding rates, among others. Furthermore, they must also form expectations about the unemployment rate at horizons beyond the fourth quarter ahead. We solve both of these issues by imposing a parametric model of beliefs to generate beliefs of variables for which expectations data are lacking and extrapolate unemployment forecasts beyond the fourth quarter horizon.

[10] It is easy to extend the estimation to incorporate imperfect expectations into the decision problems of other economic agents.

[11] We are currently working on incorporating SPF data for other variables into our framework.

As Angeletos, Huo and Sastry (2021) point out, most popular models of belief formation generate either under-reaction or over-reaction at all horizons. This pattern is very clearly seen in the impulse response of forecasts observed in Figure 1. To capture the estimated pattern of initial under-reaction followed by delayed over-reaction, we combine noisy information with diagnostic expectations and long memory. In doing so, we build on Bordalo et al. (2020) (who combined noisy information with standard diagnostic expectations) and on Bianchi et al. (2021) (who introduced diagnostic expectations with long memory). In Appendix D.1, we discuss the merits of this model of beliefs relative to other popular models in the literature. As Figure D.1 shows, having both features is essential to match the estimated pattern.

As we discuss in Appendix B.3, the noisy-information and long-memory diagnostic expectations model implies that the time t average expectation to a deterministic shock takes the following form:

$$\overline{E}_t[dX_{t+h}] = \underbrace{\left[(1+\theta)\frac{t+1}{\tau_\epsilon/\tau_\nu+t+1} - \theta\sum_{j=1}^{t}\alpha_j\left(\frac{t+1-j}{\tau_\epsilon/\tau_\nu+t+1-j}\right)\right]}_{\equiv\lambda_t} dX_{t+h}, \tag{55}$$

where $\overline{E}_t[X_{t+h}]$ denotes the average expectation and $\mathbb{E}_t[X_{t+h}]$ denotes the full-information and rational expectations in that same economy, and uses the following convention

$$\sum_{j=1}^{t}\alpha_j\left(\frac{t+1-j}{\tau_\epsilon/\tau_\nu+t+1-j}\right) = 0$$

if $t = 0$. This model features several parameters: θ denotes the degree of belief over-reaction, $\alpha_j \geq 0$ for $j \geq 1$ denote the memory weights and satisfy $\sum_{j=1}^{\infty}\alpha_j = 1$, and $\tau \equiv \tau_\epsilon/\tau_\nu$ denotes the ration of the precision of priors to the precision of the noisy signals.

Table 1: Belief parameters

Parameter	Description	Value
θ	Diagnostic expectation param	4.332
τ	Noisy information param	10.304
α	Long memory param 1	7.536
β	Long memory param 2	24.907

This model nests four known models as special cases. First, assume that $\tau = 0$ and $\alpha_1 = 1$. Then, this model collapses to the standard diagnostic expectations, as in Bordalo, Gennaioli and Shleifer (2018). Second, maintaining the assumption that $\tau = 0$ but allowing for the memory weights to assign mass to further away expectations, our model also nests the long-memory diagnostic expectation model used in Bianchi, Ilut and Saijo (2021). Third, assuming that $\theta = 0$ but $\tau > 0$, this model collapses to the standard noisy-information and rational expectations model as in Angeletos and Huo (2021). Finally, allowing $\theta > 0$ and $\tau > 0$ but assuming that $\alpha_1 = 1$, then this model collapses to the standard noisy-information and diagnostic expectations model used in Bordalo, Gennaioli, Ma and Shleifer (2020). Our model is best understood as extending this final model to allow for long-memory, which turns out to be essential in capturing the pattern of initial under-reaction followed by delayed over-reaction which can be seen in Figure 1.[12]

As in Bianchi, Ilut and Saijo (2021), we assume that the α_j are determined by a Beta-binomial distribution with parameters α and β. This assumption implies that we have four parameters to calibrate in this model θ, τ, α, and β. We calibrate these parameters so that the beliefs that they would imply for the unemployment rate forecasts line up with those that we observe in the data. However, note that, in solving the model, we actually use directly the observed unemployment rate forecasts and not the ones implied by this model.

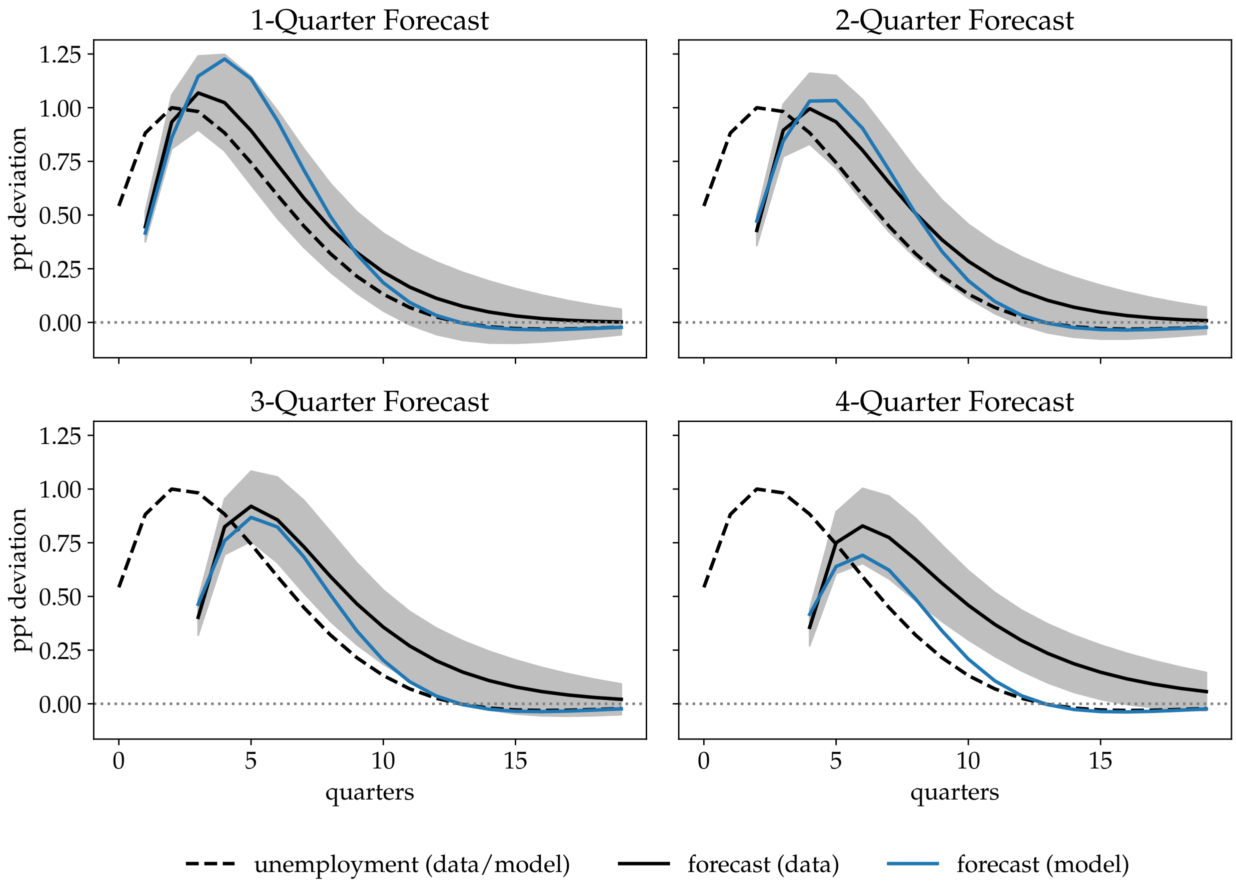
The calibrated parameters are found in Table 1 and the models empirical match can be seen in Figure 2. Overall, the fit to the data we actually observe is good. Note that, in this model, the ratio of under or over-reaction,

$$\frac{\overline{E}_t[dX_{t+h}]}{dX_{t+h}} = \lambda_t,$$

is constant across horizons. As it turns out, to match the data, the implied forecasts slightly exagerate the amount of over-reaction at the shortest horizon while underestimating the amount of over-reaction at longer horizon. We leave a more in depth analysis of this interesting fact for future work.

[12]See Appendix D.1 for a discussion.

Figure 2: Calibration of belief parameters



5 Results

In this section, we present the main results of our estimation exercise. Furthermore, we compare the implied impulse responses in our model to the benchmarks of perfect and no anticipation of future changes. This exercise allows us to understand the implications of the patterns of imperfect expectations observed in the data.

5.1 Estimation results

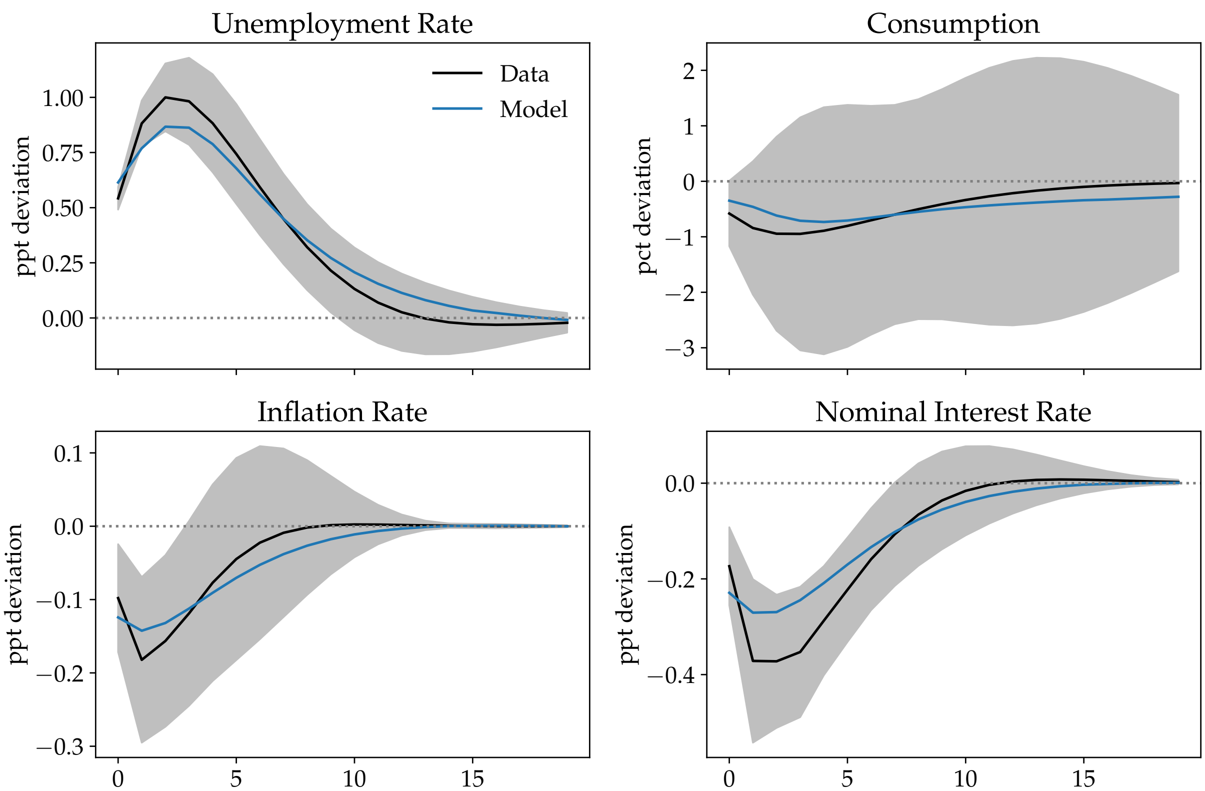
We estimate the remaining parameters which are relevant for the transition dynamics in our economy. These parameters are as follows. The monetary policy parameters: the Taylor-rule coefficient on inflation, ϕ_π, the Taylor-rule coefficient on unemployment, ϕ_u, and the Taylor-rule inertia, ρ_m. The investment adjustment cost, ψ, and the real-wage adjustment cost, ψ_w. The elasticity of the tax rate to debt, ϕ_B. The nominal rigidity parameters: price indexation, ι_p, and the slope of the Phillips curve, κ_p. The financial income payout rate, ϕ_N. Finally, the parameters controlling the scale and the persistence of the MEI shock: μ_0 and ρ_μ, respectively. The estimated values for the parameters which affect the transition dynamics in our economy can be found in Table 2.

Table 2: Estimated parameters

Parameter	Description	Value
Core model parameters		
ϕ_π	Taylor rule coef on inflation	1.241
ϕ_u	Taylor rule coef on unemployment	0.122
ρ_m	Taylor rule inertia	0.000
ψ	Investment adjustment cost	1.788
ϕ_B	Response of tax rate to debt	0.054
ψ_w	Real wage adjustment cost	1082.0
ι_p	Price indexation	0.249
κ_p	Phillips curve slope	0.075
ϕ_N	Financial income payout rate	0.009
MEI shock process		
μ_0	Scale of MEI shock	0.025
ρ_μ	Persistence of MEI shock	0.716

The model's impulse response functions are shown in the blue lines in Figure 3 for the unemployment rate, consumption, inflation rate, and the nominal interest rate. The black line shows the associated empirical impulse responses the the shaded region plots the 68% confidence interval around the empirical point estimates. The model provides a good fit to its targeted empirical counterparts.

Figure 3: Targeted Impulse Responses: Outcomes



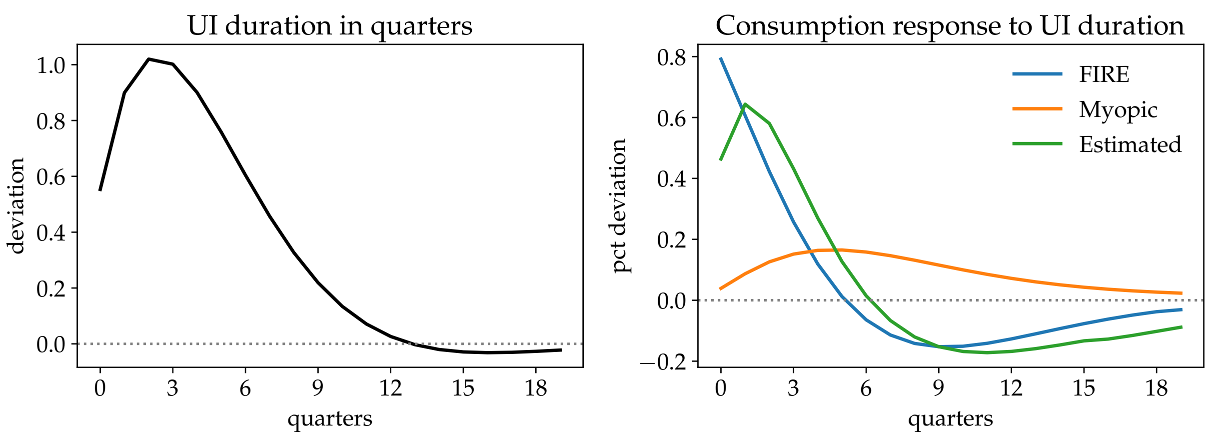
5.2 Quantifying the consequences of imperfect expectations

In this section, we assess the efficacy of UI extensions on stimulating demand with imperfect expectations. The goal is to quantify the role of imperfect anticipation of endogenous UI extensions in affecting aggregate demand. As we have discussed before, we do so by working directly with the empirical response of expectations, avoiding the need to choose a particular model of belief formation. We show that the direct effect of UI extensions on the distribution of income is less important than their indirect effect on precautionary saving. This implies that the power of UI extensions to boost aggregate demand is diminished if households do not anticipate them. We show this result in partial equilibrium (using only the calibrated household block) as well as in general equilibrium (using the full estimated HANK model).

Partial-equilibrium analysis. UI extensions can boost aggregate demand by two channels. First, directly, by raising the income of unemployed households who get to keep their benefits thanks to the extension. Second, indirectly, by reducing the precautionary savings of employed households facing the risk of job loss, and of unemployed households facing the risk of losing benefits. Our first goal is to establish that the precautionary saving channel is quantitatively relevant.

We consider a UI extension that would be triggered, according to policy rule (42), by the empirical impulse response of unemployment with respect to the main business cycle shock. The path of UI duration is plotted in the left panel of Figure 4. We feed this path of UI duration to the households of our HANK model and compute the response of aggregate consumption under different assumptions about expectations. For the purposes of this partial equilibrium exercise, we keep all other prices, income, and the job-finding rate constant at their steady-state level.[13] We contrast the response of the economy under the estimated beliefs with two extreme benchmarks: (1) full-information and rational expectations (FIRE) and (2) myopia. The first benchmark assumes that people have the correct expectations, which implies that they make no forecast errors. The second benchmark assumes that people never revise their beliefs about the future and so consistently make forecast errors. So the first benchmark features perfect anticipation of UI benefits, while the second benchmark features no anticipation of UI benefits.

Figure 4: Partial-equilibrium Consumption Responses to an UI Extension



The right panel of Figure 4 shows the impulse response of aggregate consumption. The blue line is computed assuming that households have FIRE or perfect foresight of the rise in UI duration from period 0 onwards. In this scenario, aggregate consumption rises sharply on impact due to an immediate reduction in precautionary savings, stays above steady state for five quarters, and then falls below steady state as households start to build back their normal buffer stock of savings. To understand why consumption falls below steady state before it recovers, note that a UI extension raises incomes only for those workers that lose their job and stay eligible longer. The majority of households remain continuously employed during the period of the UI extension. From their perspective, UI extension reduces risk, but provides no income. They optimally adjust their buffer stock in response to the change in unemployment risk.

[13] That is, our results depend only on the calibrated household block, and are independent from the supply side and policy blocks of the model.

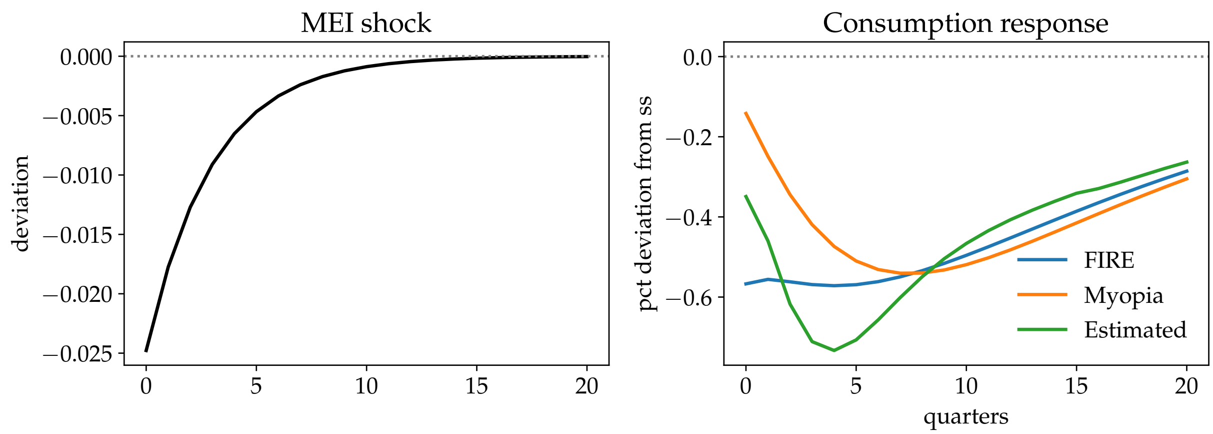
The second scenario, myopia, isolates the role of actual transfers, as household (wrongly) forecast no change to their income prospects upon unemployment. The orange line shows that the resulting response of aggregate consumption is markedly different from the first scenario with FIRE. Conditional on their individual states, the households' consumption-saving decisions do not change at all. The hump-shaped aggregate consumption response is driven entirely by changes in the distribution. The mass of UI eligible households rises while the mass of ineligible unemployed households falls. Aggregate consumption rises moderately because the households who receive UI benefits consume more on average than those who exhausted their benefits. The comparison of this scenario with FIRE demonstrates the importance of anticipation of UI benefits in shaping the consumption response to the policy. In fact, the peak response of aggregate consumption to unemployment benefits is over four times as large with FIRE than with myopia, and it happens on impact as opposed to 5 quarters later.

In the third scenario, we give households the expectations estimated in the ARMA-IV regression (54). As figure 1 shows, estimated beliefs feature initial dampening followed by delayed over-reaction relative to the actual path of the unemployment rate. Since we assume that households have first-order knowledge of the policy, the same pattern applies to beliefs about UI duration. It follows that the initial response of aggregate consumption is muted relative to FIRE, but much higher than the myopic scenario since people still anticipate some of the UI extension. However, because of the over-reaction in beliefs, after a few periods the response of aggregate demand becomes even higher than under FIRE due to the effect of perceived UI duration on precautionary savings.

General-equilibrium analysis. We established that imperfect anticipation of UI extensions has a large impact on the partial-equilibrium response of aggregate demand to the policy. Next, we compute the consequences imperfect anticipation in the full dynamic general-equilibrium model.

Figure 5 displays the impulse response of aggregate consumption to the marginal efficiency of investment (MEI) shock. As in the previous section, we compare the response in our baseline economy with the estimated beliefs to the benchmarks of perfect anticipation (FIRE) and no anticipation (Myopia). In performing these comparisons, we fix all parameters (other than those relating to beliefs) to their estimated values (see section 4.6). We then compute the dynamic response of those benchmarks to the same MEI shock. The response of our baseline economy can be seen in green, while the response under FIRE and Myopia can be seen in blue and orange.

Figure 5: General Equilibrium Impulse Responses to MEI Shock



Aggregate consumption falls in response to this negative MEI shock for all models, mostly because firms invest less and hire less workers, leading to a rise in unemployment and a decline in incomes. As unemployment surges the government responds by increasing UI benefits, which helps stimulate the economy, but does not fully offset the shock.

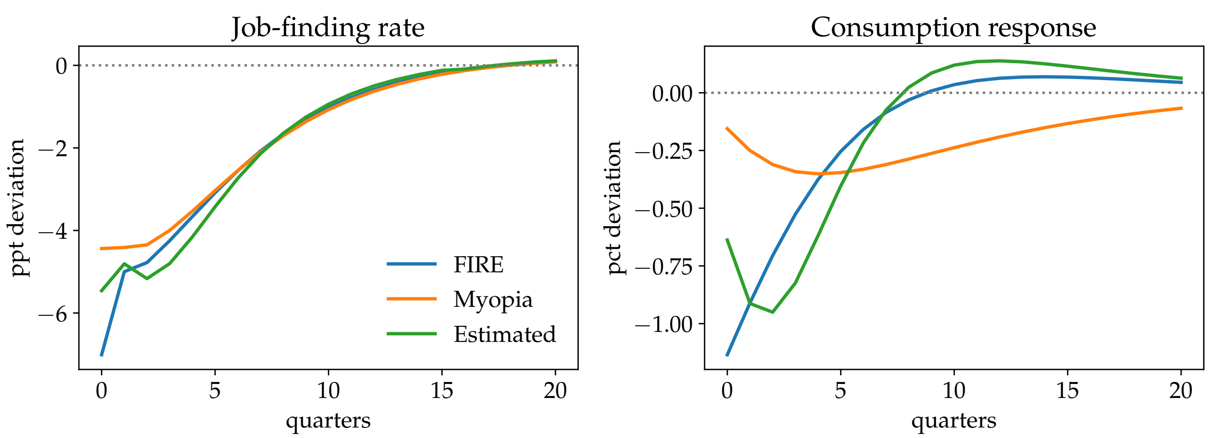
The initial drop in consumption is less pronounced in the baseline model than with FIRE. This result is a consequence of the fact that individuals are more optimistic about the depth of the recession due to the initial under-reaction of beliefs, i.e., individuals think that unemployment will not rise as much. The same holds for the comparison of Myopia to the two other lines. The initial drop in consumption is -0.39, -0.58, and -0.16 percent for the baseline, FIRE, and Myopia economies.

However, after this initial period, individuals become more pessimistic about the future path of unemployment and job finding prospects than they would under FIRE or Myopia. It follows that individuals predict larger unemployment risk and so, despite also predicting higher UI benefits, they have a higher precautionary-savings motive and cut their consumption by more relative to FIRE and Myopia. These effects imply a hump-shaped response of aggregate consumption which would not be present with FIRE. The peak response of aggregate consumption with the estimated beliefs if -0.81, while for FIRE it is equal to the initial response -0.58. Over time, these very pessimistic expectations are not realized and individuals consume their excess savings, justifying the fact that consumption is higher after 10 quarters under the estimated beliefs than under both other benchmarks.

Figure 5 highlights the importance of expectations on the general-equilibrium response of aggregate consumption. However, it does not allow us to understand the independent effects of each general-equilibrium channel. To better understand the consequences of the fall in job finding rates and the endogenous rise in UI duration, we now decompose the overall GE effect. We isolate the effect of different channels on consumption, by taking the Jacobian with respect to that input and multiplying it by the impulse response of that input. We focus primarily on understanding the effects coming through these two channels due to their central importance in our analysis. In appendix D.2, we complement the analysis here by describing the effects of the remaining GE forces.

Job-finding rate. As a consequence of the MEI shock, firms post less vacancies which leads to a decline in the job-finding rate. In Figure 6, we evaluate the effect of this general-equilibrium channel on aggregate consumption for our baseline economy (Estimated) and the two benchmarks of perfect anticipation (FIRE) and no anticipation (Myopia). The left panel shows the IRF for the job-finding rate while the panel on the right computes the isolates the effect of the decline in the job-finding rate for aggregate consumption.

Figure 6: Partial effect of job-finding rate on consumption



The impulse responses for the job-finding rate are very similar across the three models. The initial drop is larger under FIRE and smaller with myopic beliefs. The baseline economy sees an initial smaller drop in the job finding rate relative to FIRE, but the ranking is reversed after 3 periods.

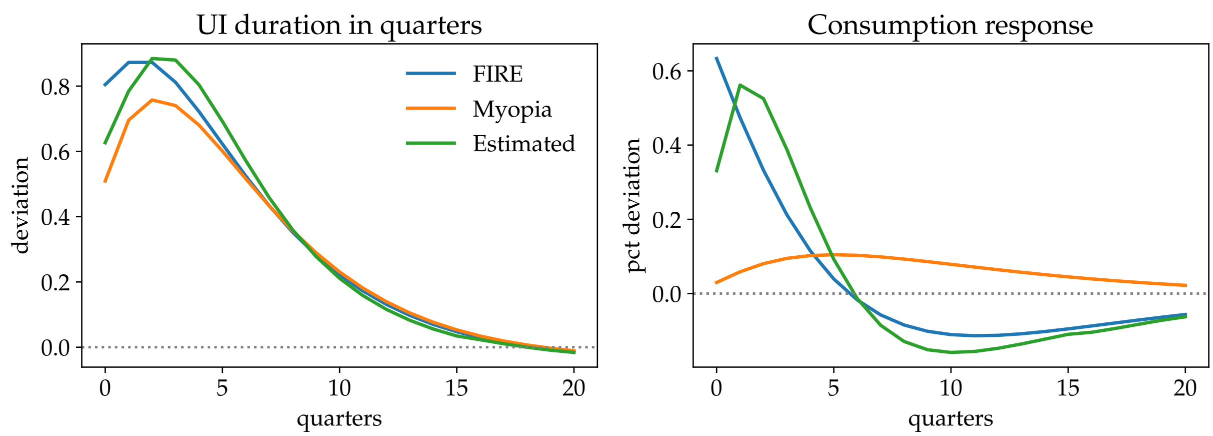
Despite featuring similar job-finding rates, there are large differences in the response of aggregate consumption. With myopic beliefs, since people do not anticipate the coming recession, the initial decline in consumption is small. However, as unemployment rises, workers are faced with unexpectedly longer unemployment spells which leads to a severe contraction of spending in later periods. Instead, with FIRE, there is a large initial drop in consumption which fully recovers by period 10.

In our baseline economy, we see that because beliefs initially under-react, the consumption response is muted relative to FIRE, but it is larger than under myopia. However, after this initial phase, individuals become more pessimistic and the average belief over-reacts relative to FIRE.

The consequence is a pronounced drop in aggregate consumption. It is noteworthy that this pattern of delayed over-reaction in beliefs implies a hump-shaped response of consumption to the job-finding rate, which would not be possible under FIRE.

UI duration extension. As the job-finding rate drops, the unemployment rate rises which triggers an extension in the duration of unemployment insurance. In Figure 7, we evaluate the effect of this general-equilibrium channel on aggregate consumption for our baseline economy (Estimated) and the two benchmarks of perfect anticipation (FIRE) and no anticipation (Myopia). The left panel shows the IRF for the UI duration while the panel on the right computes the isolates the effect of the increase in UI duration for aggregate consumption.

Figure 7: Partial effect of UI duration extension on consumption



The duration of UI tracks exactly the impulse response of the unemployment rate, which follows a hump shape. Unemployment rises initially the most for FIRE and the least for myopic beliefs. The baseline economy features an initial lower unemployment rate than FIRE, but a larger peak level of unemployment. These dynamics feed exactly to the UI duration.

As we have discussed before, extending UI stimulates consumption for two reason: the redistribution channel and the precautionary savings channel. The panel on the right shows that, under myopic beliefs, the impact of UI extensions on aggregate consumption is very small when compared to FIRE. This shows that the major source of stimulus comes from the precautionary savings channel rather than the redistribution channel. With FIRE, UI extensions are very powerful at stimulating consumption especially on impact. The strength of the initial impact is largely a consequence of the anticipation of higher future UI benefits, leading people to save less and thus consume more.

Our baseline economy features lower initial anticipation of UI extensions relative to the FIRE benchmark. Figure 7 shows that this initially mutes the consumption response to UI extensions.

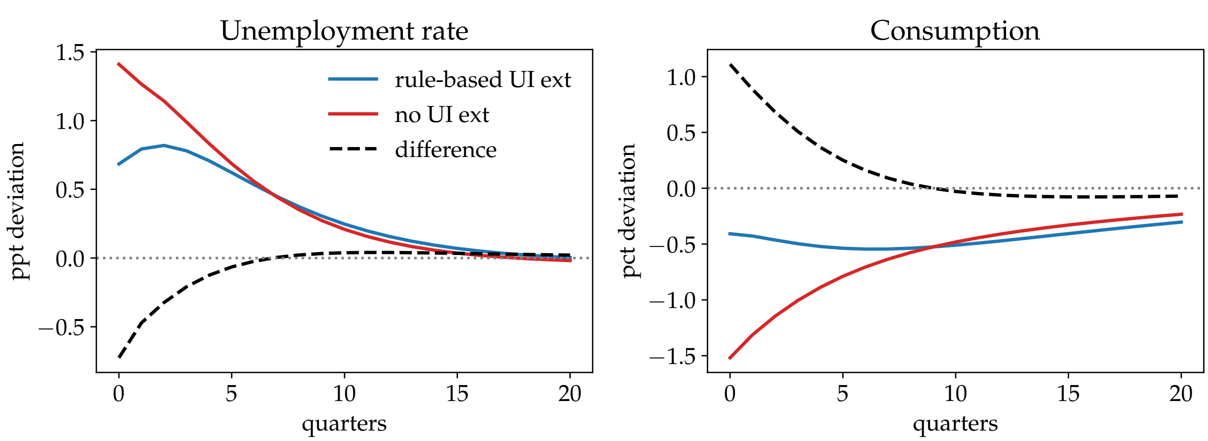
However, as noted before, the empirical pattern of beliefs implies that eventually beliefs overreact relative FIRE. It follows that they begin anticipating larger UI benefits and thus a stronger stimulative power for this policy.

6 Quantifying the stimulative power of UI extensions

The results in the previous section allow us to understand the importance of anticipating UI extensions in determining their efficacy in stimulating consumption. Furthermore, they allow us to compare the implications of the empirical patters of beliefs relative to two important benchmarks: FIRE and myopia. However, those results do not allow us to quantify the power of UI extensions. For that purpose, we need to compare the impulse responses obtained in the previous section with those obtained in a counterfactual economy assuming no extensions, i.e., $\zeta_b = 0$.

For the purposes of computing a counterfactual response, we must take into account how beliefs differ in this counterfactual economy relative to our baseline analysis. This means that we must specify a model for belief formation, and can no longer simply rely on the estimated beliefs. We assume that beliefs are determined by the noisy-information and long-memory diagnostic expectations model (baseline) that we estimate in Section 4.6. This choice is in line with our discussion in that section and further extended in Appendix D.1.

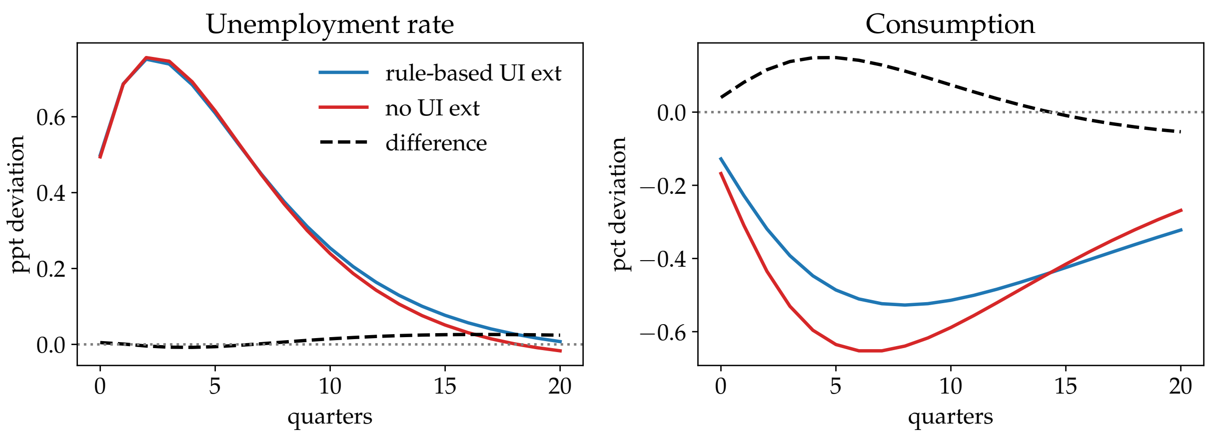
Figure 8: Impact of UI extension with FIRE



Figures 8, 9, and 10 display the impulse responses under FIRE, myopia, and baseline beliefs, respectively. For each figure, the left panel displays the impulse responses for the unemployment rate with the policy (blue) and without the policy (red). The black dashed line plots the difference between these two responses and is a measure of the stimulus. The right panel displays the analogous three responses for consumption.

Figure 8 shows that, with FIRE, the strongest effects of UI extensions happen immediately on impact of the shock. At this initial date, the policy leads to a decrease in the unemployment rate of 0.7 percentage points and in increase in consumption of 1 percentage point on impact. The stimulus effect then declines over time and dissipated by the eighth quarter.

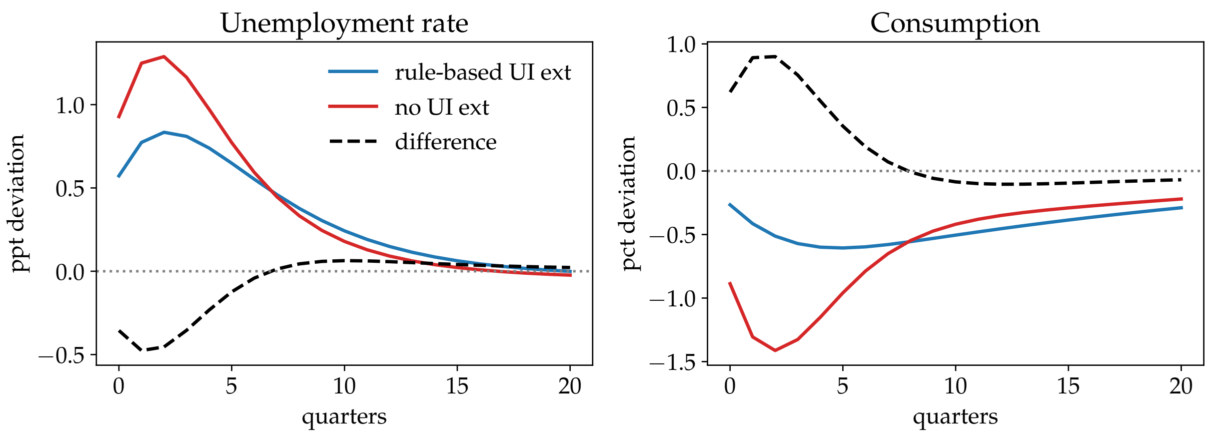
Figure 9: Impact of UI extension with Myopia



The positive impact of UI extensions are mostly a consequence of the effect that their anticipation has on precautionary savings. This fact is most clearly seen in the comparison of the efficacy of UI extensions with FIRE (Figure 8) and Myopia (Figure 9). In fact, with Myopia we see that the extensions have almost no impact on the unemployment rate and only a very moderate impact on consumption (echoing the partial equilibrium results in the previous section). We see that the peak impact of UI extensions is slightly delayed relative to FIRE, which is a consequence of the fact that without anticipation, only the redistribution channel is operative and so the impact of the policy tracks closely the UI duration at each point in time. Still, the peak impact of UI extensions is essentially zero for the unemployment rate and less than 0.1 percentage points for consumption.

Figure 10 shows the analogous results for the baseline economy. We can see several features. First, the impact of UI extensions are dampened relative to FIRE. This feature is especially true at the onset of the shock, where the promise of UI extensions are responsible for a decrease in the unemployment rate of close to 0.4 percentage points and an increase in consumption of 0.6 percentage points. Due to the delayed over-reaction of beliefs, the impact of UI extensions also follows a hump shape. Intuitively, in the economy with delayed belief over-reaction, individuals switch from their initial over-optimism to over-pessimism about the future unemployment rate. However, since people understand the policy rule, they believe that this increase in unemployment will trigger an expansion of UI generosity. The peak impact of UI extensions leads decreases the unemployment rate by over 0.5 percentage points and increases consumption by close to 1 percentage point.

Figure 10: Impact of UI extension in the baseline economy



7 Alternative policy implementation

In this section, we are interested in understanding how the efficacy of UI duration extensions is affected by the way in which the policy is implemented. As in Bianchi-Vimercati, Eichenbaum and Guerreiro (2021), we are interested in comparing the impact of the policy when it is implemented and announced as a rule versus when the path of UI duration is directly announced. In the latter case, we assume that the government directly announces a path for the policy variable π_t^{lose} and that this announcement is immediately learned and understood by all market participants.

As in Section 6, making this comparison requires us to compute a counterfactual path for the economy under a different policy implementation. So in our baseline economy, we maintain the assumption that beliefs are given by the noisy-information and long-memory diagnostic expectations model (baseline) that we estimate in Section 4.6. We first compute the response of the economy under the assumption that the policy is implemented as a rule. We then recover the implied path for UI duration and compute the dynamic response in the counterfactual economy where the same policy is directly announced at the onset of the recession. As in the previous section, we do this analysis in our baseline economy and for comparison also perform the analysis under FIRE and myopia.

Figures 11, 12, and 13 display the impulse responses under FIRE, myopia, and baseline beliefs, respectively. For each figure, the left panel displays the impulse responses for the unemployment rate with the rules-based policy (blue) and with the instrument-announcement policy (brown). The black dashed line plots the difference between the equilibrium with announcement and that with rules. The right panel displays the analogous three responses for consumption.

Figure 11: Instrument-rule vs announcement with FIRE

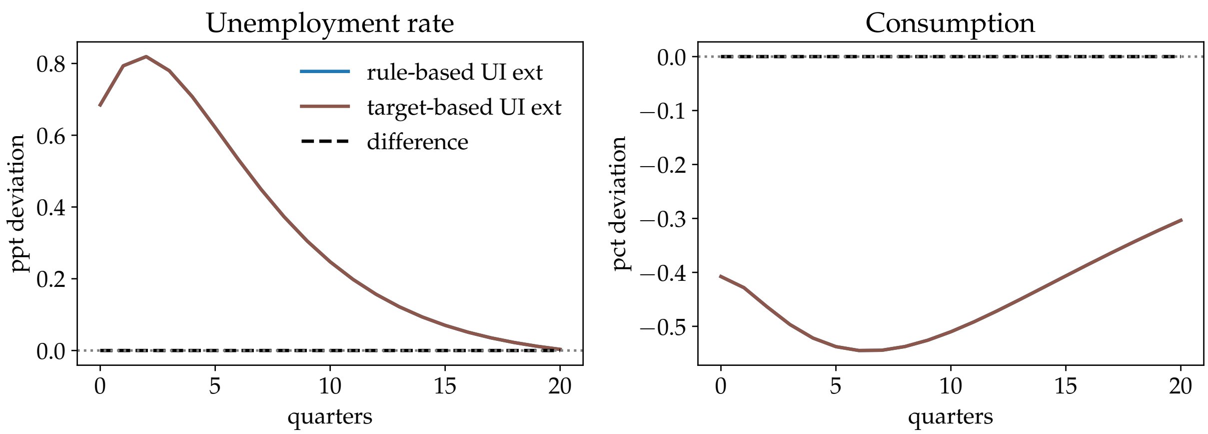
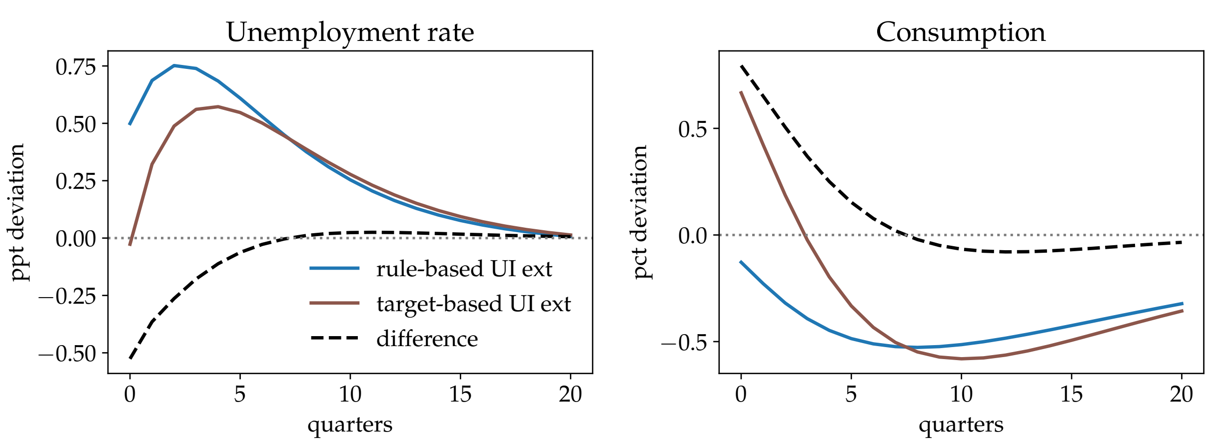


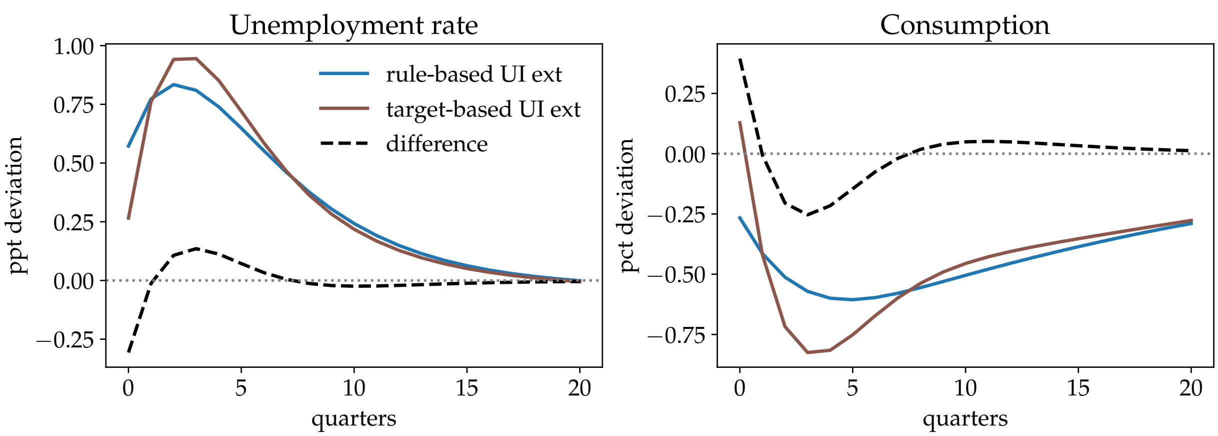
Figure 11 shows that with perfect anticipation, i.e., with FIRE, the two forms of policy implementation lead to the same outcomes. With FIRE, it doesn't make any difference whether the instruments are directly announced or that they are announced as a rule, since people accurately forecast the behavior of the unemployment rate and can thus use it to perfectly predict the future path of unemployment duration (see also Angeletos and Sastry, 2021, and Bianchi-Vimercati, Eichenbaum and Guerreiro, 2021). Instead, Figure 12 shows that with myopia there can be large differences between these two forms of implementation. When the instrument is directly announced to people, the model features large anticipation of UI benefits and so a strong stimulative power of the policy. Indeed, the unemployment rate falls by over 0.5 percentage points on impact and consumption is boosted by almost 0.7 percentage points on impact.

Figure 12: Instrument-rule vs announcement with myopia



Finally, in our baseline economy, the results are mixed. The announcement-based policy limits the initial rise in the unemployment rate by over 0.25 percentage points, and the fall in consumption by 0.28 percentage points. After the initial period, the order is reversed and the unemployment rate becomes 0.15 percentage points higher in the economy with the announcement-based policy. Similarly, consumption falls 0.25 percentage points more. These results are a direct consequence of the pattern of delayed over-reaction present in beliefs. Initially, beliefs under-react, so people under-forecast the generosity of UI extensions in the economy with the policy rule. So it is more powerful to directly announce the extension in UI benefits.[14] However, after this initial period, beliefs over-react and people become overly pessimistic. So under the rule-based policy, people believe that UI benefits will be extended for longer. In other words, the rule-based policy exploits the belief over-reaction and stimulates demand further without an actual increase in UI duration.

Figure 13: Instrument-rule vs announcement in the baseline economy



We conclude that in our baseline economy, changing the implementation of UI extensions to a direct announcement could help their stimulative power at the onset of the recession, but may lack efficacy past the peak of the recession when beliefs turn overly pessimistic.

8 Conclusion

Economists have long emphasized the benefits of linking UI benefits duration to aggregate economic conditions (see, e.g., Chodorow-Reich and Coglianese 2019, Eichenbaum 2019, Mitchell and Husak 2021). In this paper, we argue that expectations are critical in determining the stabilization power of these policies.

[14] This argument follows the same logic as in Bianchi-Vimercati et al. (2021).

We study the economic impact of UI extensions in a state-of-the-art Heterogeneous Agent New Keynesian model with search and matching frictions. We discuss a general framework to solve and analyze such models under arbitrary beliefs about macroeconomic outcomes. We leverage the framework to estimate the model to match the impulse responses of key aggregate variables and expectations to identified business-cycle shocks. By doing so, we demonstrate that expectations data can be used directly to solve the model, thus sidestepping the issue of choosing among the "wilderness" of alternative models for belief formation. Our results emphasize that the stimulative power of state-dependent UI extensions can be greatly affected by systematic forecast errors that people make in predicting the business cycle.

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A Appendix to section 2

A.1 Deriving the equilibrium

We solve the model backwards, starting at the end of period 1. At this point, households observe the relevant equilibrium outcomes $\{b_1, \tau_1\}$ as well as their employment status $e_1 \in \{E, U\}$. Their problem is

$$V_1(e_1, a_0) = \max_{c_1, a_1} u(c_1) \qquad \text{s.t. } c_1 + a_1 = (1+r)a_0 + \mathbf{1}_{\{e_t = E\}} + \mathbf{1}_{\{e_1 = U\}} \cdot b_1 - \tau_1 \qquad a_1 \geq 0 \tag{A.1}$$

where we already imposed $w_1 = 1$. Clearly, the optimal decision is to consume the entire cash on hand

$$a_1(e_1, a_0) = 0 \tag{A.2}$$

$$c_1(e_1, a_0) = (1+r)a_0 + \mathbf{1}_{\{e_1 = E\}} + \mathbf{1}_{\{e_1 = U\}} \cdot b_1 - \tau_1 \tag{A.3}$$

Since assets are in zero net supply and borrowing is not allowed, all workers have zero assets in equilibrium, $a_0 = 0$. Given the policies $\{b_1, M_1\}$, the time-1 equilibrium can be computed recursively as

$$N_1 = M_1 \tag{A.4}$$

$$\tau_1 = (1 - N_1)b_1 \tag{A.5}$$

$$c_1(E) = 1 - \tau_1 \tag{A.6}$$

$$c_1(U) = b_1 - \tau_1 \tag{A.7}$$

Note that time-1 equilibrium is independent of what happens in period 0, including the beliefs $\{N_1^e, b_1^e, \tau_1^e\}$ that households hold in period 0.

Let's turn to period 0. Combining the consumption policy function (A.3) with the fact that the probability of employment is iid, the expected continuation value at the end of period 0 can be written as

$$V_1^e(a_0) = N_1^e \cdot u\Big((1+r)a_0 + 1 - \tau_1^e\Big) + (1 - N_1^e) \cdot u\Big((1+r)a_0 + b_1 - \tau_1\Big) \tag{A.8}$$

At time $t = 0$, households solve

$$\max_{c_0, a_0} u(c_0) + \beta V_1^e(a_0) \qquad \text{s.t. } c_0 + a_0 = \mathbf{1}_{\{e_0 = E\}} + \mathbf{1}_{\{e_0 = U\}} \cdot b_0 - \tau_0 \qquad a_0 \geq 0 \tag{A.9}$$

Taking FOCs yields the Euler equation (5) in the main text

$$u'(c_0) \geq \beta (V_1^e)'(a_0) \tag{A.10}$$

$$= \beta(1+r)\Big[N_1^e \cdot u'\Big(1-\tau_1^e+(1+r)a_0\Big)+(1-N_1^e)\cdot u'\Big(b_1^e-\tau_1^e+(1+r)a_0\Big)\Big] \tag{A.11}$$

Note that the right-hand side does not depend on time-0 employment status. Since there can't be any saving in equilibrium ($a_0 = 0$), households consume their income in period 0, which is higher for employed households. Given that $u'(\bullet)$ is decreasing, this implies that either both types of households are borrowing constrained or only unemployed workers are constrained. We assume that β and r is such that only unemployed households are constrained.

Then, given beliefs $\{N_1^e, \tau_1^e, b_1^e\}$ and policies $\{b_0, r\}$, the time-0 equilibrium can be computed recursively as follows

$$u'\Big(c_0(E)\Big) = \beta(1+r)\Big[N_1^e \cdot u'\Big(1-\tau_1^e\Big)+(1-N_1^e)\cdot u'\Big(b_1^e-\tau_1^e\Big)\Big] \tag{A.12}$$

$$\tau_0 = 1 - c_0(E) \tag{A.13}$$

$$c_0(U) = b_0 - \tau_0 \tag{A.14}$$

$$N_0 = 1 - \frac{\tau_0}{b_0} \tag{A.15}$$

$$M_0 = N_0 \tag{A.16}$$

A.2 Proof of proposition 1

Proof. Consider an infinitesimal shock to nominal GDP in period 1, dM_1. Differentiating the time-1 equilibrium (A.12)–(A.16) yields

$$dN_1 = dM_1 \tag{A.17}$$

$$d\tau_1 = (1-N_1)db_1 - dN_1 \cdot b_1 \tag{A.18}$$

$$dc_1(E) = -d\tau_1 \tag{A.19}$$

$$dc_1^U = db_1 - d\tau_1 \tag{A.20}$$

Since we assumed that both UI extension regimes implement the same benefits db_1, the time-1 responses are the same under both regimes. Given our model of beliefs (10), this implies that expectations of employment and taxes are the same

$$dN_1^{e,rule} = dN_1^{e,*} = \lambda \cdot dN_1 \qquad \text{and} \qquad d\tau_1^{e,rule} = d\tau_1^{e,*} = \lambda \cdot d\tau_1 \tag{A.21}$$

but the expectation of unemployment benefits may differ

$$db_1^{e,rule} = -\zeta_b \cdot dN_1^{e,rule} = -\zeta_b \lambda \cdot dN_1 \leq -\zeta_b \cdot dN_1 = db_1^{e,*} \tag{A.22}$$

These expectations are relevant for pinning down $dc_0(E)$ through the Euler equation of employed workers (A.12). To first order after the shock, the Euler equation reads as

$$u''(1-\tau_0)\cdot dc_0(E) = \beta(1+r)\Big[dN_1^e\cdot u'\left(1-\tau_1^e\right) - N_1^e\cdot u''\left(1-\tau_1^e\right)d\tau_1^e \\ - dN_1^e u'\left(b_1^e-\tau_1^e\right) + (1-N_1^e)\cdot u''\left(b_1^e-\tau_1^e\right)(db_1^e - d\tau_1^e)\Big] \tag{A.23}$$

So the difference in consumption under the two UI extension regimes is

$$dc_0(E)^{\text{rule}} - dc_0(E)^{\text{ann}} = \underbrace{\frac{\beta(1+r)(1-N_1^e)\cdot u''(b_1^e-\tau_1^e)}{u''(1-\tau_0)}}_{\equiv M_b}\cdot\left(db_1^{e,rule} - db_1^{e,*}\right) \tag{A.24}$$

where $M_b\in[0,1]$ can be interpreted as the marginal propensity to consume out of anticipated UI benefits. Note that the dN_1^e and $d\tau_1^e$ terms cancel because these expectations are independent of the UI extension regime.

Differentiating the rest of the time-0 equilibrium conditions (A.13)–(A.16) gives us

$$d\tau_0 = -dc_0(E) \tag{A.25}$$

$$dc_0(U) = db_0 - d\tau_0 \tag{A.26}$$

$$dN_0 = -\frac{d\tau_0}{b_0} + \frac{\tau_0}{b_0^2}db_0 \tag{A.27}$$

$$dM_0 = dN_0 \tag{A.28}$$

Let's assume that UI benefits respond only in period 1, $db_0 = 0$, in order to isolate the impact of precautionary behavior. Combining the perturbed time-0 equilibrium conditions proves the proposition

$$dY_0^{\text{rule}} - dY_0^{\text{ann}} = \frac{1}{b_0}\cdot M_b\cdot(1-\lambda)\cdot dM_1 \tag{A.29}$$

To interpret the $1/b_0$ term, note that the aggregate consumption function of this economy is

$$C_0 = N_0\cdot c_0(E) + (1-N_0)(b_0-\tau_0) \tag{A.30}$$

According to (A.12), the consumption choice of employed workers $c_0(E)$ does not depend on N_0. Also recall that, in equilibrium, $c_0(E) = 1-\tau_t$. So

$$\frac{\partial C_0}{\partial N_0} = c_0(E) - (b_0-\tau_0) = 1 - b_0$$

This implies that

$$\frac{1}{b_0} = \frac{1}{1-\frac{\partial C_0}{\partial N_0}} \equiv \mathcal{M} > 0 \tag{A.31}$$

is a standard Keynesian multiplier. □

B Appendix to section 3

B.1 Proof of Proposition 2

We consider a generic representation of a heterogeneous-agent problem as a mapping from some input X_t to a time-path of aggregates C_t. Following Auclert et al. (2021), a generic representation of a heterogeneous-agent problem is a mapping between aggregate inputs $\boldsymbol{X}_t$, a time path f aggregate outputs $\boldsymbol{Y}_t$. Assume that there are n_x inputs and n_y outputs, and that the distribution if discretized on n_g points. Let $\boldsymbol{D}_t$ denote the $n_g \times 1$ distribution of agents. Then let $\boldsymbol{y}_t$ be the $n_g \times n_y$ matrix of individual outcomes.

$$\boldsymbol{v}_t = v\left(\boldsymbol{v}_{t+1}^e, \boldsymbol{X}_t\right) \tag{B.1}$$

$$\boldsymbol{v}_t^e = v\left(\boldsymbol{v}_{t+1}^e, \boldsymbol{X}_t^{e,0}\right) \tag{B.2}$$

$$\boldsymbol{D}_{t+1} = \Lambda\left(\boldsymbol{v}_{t+1}^e, \boldsymbol{X}_t\right)' \boldsymbol{D}_t \tag{B.3}$$

$$\boldsymbol{Y}_t = y\left(\boldsymbol{v}_{t+1}^e, \boldsymbol{X}_t\right)' \boldsymbol{D}_t \tag{B.4}$$

Let $(\boldsymbol{Y}, \boldsymbol{v}, \boldsymbol{v}^e, \boldsymbol{D})$ denote the steady state which satisfies $\boldsymbol{X}^e = \boldsymbol{X}$. This immediately implies that $\boldsymbol{v} = \boldsymbol{v}^e$. For convenience, let $\Lambda_{ss} \equiv \Lambda\left(\boldsymbol{v}^e, \boldsymbol{X}\right)$. Consider transitions of length T that satisfy $\boldsymbol{X}_{t-1} = \boldsymbol{X}$ and $\boldsymbol{v}_T^e = \boldsymbol{v}_T = \boldsymbol{v}$. The initial distribution $\boldsymbol{D}_0$ is given and we assume that $\boldsymbol{D}_0 = \boldsymbol{D}$.

Given all of this, this defines a $T \times n_y$ vector of stacked outputs

$$\boldsymbol{Y} = h\left(\boldsymbol{X}, \boldsymbol{X}^{e,0}\right).$$

Assume that all functions are differentiable, then so it h. We want to characterize the Jacobian $\mathcal{J}$ of h evaluated at the stead state with respect to variables X and X^e.

Responde to dX_s Consider a change to input X at time s, dX_s, with $dX_t = 0$ for all $t \neq s$. It follows immediately that

$$v_t^e = v^e = v,$$

for all t, and $v_t = v$ for all $t \neq s$. Furthermore, it follows that, for all $t \neq s$, $\boldsymbol{y}_t \equiv y\left(\boldsymbol{v}_{t+1}^e, \boldsymbol{X}_t\right) = \boldsymbol{y}$ and $\boldsymbol{\Lambda}_t \equiv \Lambda\left(\boldsymbol{v}_{t+1}^e, \boldsymbol{X}_t\right) = \boldsymbol{\Lambda}$, so $d\boldsymbol{y}_t = 0$ and $d\boldsymbol{\Lambda}_t = 0$.

Note that, by the chain rule, we find that:

$$d\boldsymbol{Y}_t = d\boldsymbol{y}_t'\boldsymbol{D} + \boldsymbol{y}d\boldsymbol{D}_t$$

and

$$d\boldsymbol{D}_{t+1} = d\boldsymbol{\Lambda}_t'\boldsymbol{D} + \boldsymbol{\Lambda}'d\boldsymbol{D}_t.$$

Using these expressions and the results above, it follows that $dY_t = 0$ and $d\boldsymbol{D}_{t+1} = 0$ for all $t < s$.

Furthermore, for $t = s$, we obtain

$$d\boldsymbol{D}_{s+1} = d\boldsymbol{\Lambda}_s' \boldsymbol{D}, \quad \text{and} \quad d\boldsymbol{Y}_s = d\boldsymbol{y}_s' \boldsymbol{D},$$

and for $t > s$ we find that

$$\boldsymbol{D}_t = \boldsymbol{\Lambda}' d\boldsymbol{D}_{t-1} = \left(\boldsymbol{\Lambda}'\right)^{t-(s+1)} d\boldsymbol{D}_{s+1}, \quad \text{and} \quad d\boldsymbol{Y}_t = \boldsymbol{y} d\boldsymbol{D}_t$$

Finally, note that $d\boldsymbol{y}_s$ does not depend on the time s, but rather than the shock happens at that moment and is not anticipated. It immediately follows that

$$\frac{\partial Y_t}{\partial X_s} = \begin{cases} 0 & \text{if } t < s \\ \frac{\partial Y_{t-s}}{\partial X_0}, & \text{if } t \geq s \end{cases} = \begin{cases} 0 & \text{if } t < s \\ \mathcal{J}_{t-s,0}, & \text{if } t \geq s, \end{cases} \tag{B.5}$$

where $\mathcal{J}$ denotes the FIRE Jacobian.

Response to dX_s^e Note that, $v_{s+t}^e = v_{t+1} = v$ for all $t \geq s$, which implies that $\boldsymbol{\Lambda}_t = \boldsymbol{\Lambda}$ and $\boldsymbol{y}_t = \boldsymbol{y}$. It follows that, for $t > s$,

$$d\boldsymbol{Y}_t = \boldsymbol{y} d\boldsymbol{D}_t$$

and

$$d\boldsymbol{D}_{t+1} = \boldsymbol{\Lambda}' d\boldsymbol{D}_t$$

where $d\boldsymbol{D}_t = \boldsymbol{\Lambda}' d\boldsymbol{D}_{t-1} = \left(\boldsymbol{\Lambda}'\right)^{t-(s+1)} d\boldsymbol{D}_{s+1}$.

So, for $t < s$, the response is exactly the same as that which would be obtained under FIRE, i.e., $dY_t = \mathcal{J}_{t,s}$. For $t = s$, we find that $v_{s+1}^e = v$ and since $X_s = X$, then $\boldsymbol{y}_s = \boldsymbol{y}$ and $\boldsymbol{\Lambda}_s = \boldsymbol{\Lambda}$. It follows that

$$d\boldsymbol{D}_{s+1} = \boldsymbol{\Lambda} d\boldsymbol{D}_s^{\text{ann}} = d\boldsymbol{D}_{s+1}^{\text{ann}} - \left(d\boldsymbol{\Lambda}_s^{\text{ann}}\right)' \boldsymbol{D} = d\boldsymbol{D}_{s+1}^{\text{ann}} - d\boldsymbol{D}_{s+1}^{0}$$

and

$$d\boldsymbol{Y}_s = \boldsymbol{y} d\boldsymbol{D}_s^{\text{ann}} = d\boldsymbol{Y}_s^{\text{ann}} - \left(d\boldsymbol{y}_s^{\text{ann}}\right)' \boldsymbol{D} = d\boldsymbol{Y}_s^{\text{ann}} - d\boldsymbol{Y}_s^{0},$$

where dD_s^* and $d\Lambda_s^*$ denote the response under FIRE, i.e., $dX_s^e = dX_s$, and dD_s^0 and $d\Lambda_s^0$ denote the responses to an unanticipated change $dX_s \neq 0$ with $dX_s^{e,0} = 0$. Finally, for $t > s$, we also find that $\boldsymbol{v}_{t+1}^e = \boldsymbol{v}$ and $\boldsymbol{X}_t = \boldsymbol{X}$ so that decisions and transitions do not change. As a result,

$$d\boldsymbol{D}_t = \left(\boldsymbol{\Lambda}'\right)^{t-(s+1)} d\boldsymbol{D}_{s+1} = \left(\boldsymbol{\Lambda}'\right)^{t-(s+1)} \left(d\boldsymbol{D}_{s+1}^{\text{ann}} - d\boldsymbol{D}_{s+1}^{0}\right) = d\boldsymbol{D}_t^{\text{ann}} - d\boldsymbol{D}_t^{0}$$

$$d\boldsymbol{Y}_t = \boldsymbol{y} d\boldsymbol{D}_t = \boldsymbol{y} d\boldsymbol{D}_t^{\text{ann}} - \boldsymbol{y} d\boldsymbol{D}_t^{0} = d\boldsymbol{Y}_t^{\text{ann}} - d\boldsymbol{Y}_t^{0}.$$

As a result,

$$\frac{dY_t}{dX_s^{e,0}} = \begin{cases} \mathcal{J}_{t,s} & \text{if } t < s \\ \mathcal{J}_{t,s} - \mathcal{J}_{t-s,0} & \text{if } t \geq s \end{cases} \tag{B.6}$$

Putting it together Define

$$\mathcal{E} \equiv \begin{bmatrix} \mathcal{J}_{0,0} & 0 & 0 & ... \\ \mathcal{J}_{1,0} & \mathcal{J}_{0,0} & 0 & ... \\ \mathcal{J}_{2,0} & \mathcal{J}_{1,0} & \mathcal{J}_{0,0} & ... \\ ... & ... & ... & ... \end{bmatrix}, \tag{B.7}$$

then we can summarize these results in the following expressions

$$d\boldsymbol{Y} = (\mathcal{J} - \mathcal{E}) \cdot d\boldsymbol{X}^{e,0} + \mathcal{E} \cdot d\boldsymbol{X}^0 = \mathcal{J} \cdot d\boldsymbol{X}^{e,0} + \mathcal{E} \cdot (d\boldsymbol{X}^0 - d\boldsymbol{X}^{e,0}). \tag{B.8}$$

B.2 Proof of proposition 3

Let $\left\{ \left\{ X_s^{e,t} \right\}_{s=t+1}^{T-1} \right\}_{t=0}^{T-1}$ denote their beliefs at each point in time, then the representation is

$$\boldsymbol{v}_s^{e,t} = v\left(\boldsymbol{v}_{s+1}^{e,t}, \boldsymbol{X}_s^{e,t}\right), \quad s = 0, ..., T-1, t = 0, ..., T-1 \tag{B.9}$$

$$\boldsymbol{v}_t = v\left(\boldsymbol{v}_{t+1}^{e,t}, \boldsymbol{X}_t\right) \tag{B.10}$$

$$\boldsymbol{D}_{t+1} = \Lambda\left(\boldsymbol{v}_{t+1}^{e,t}, \boldsymbol{X}_t\right)' \boldsymbol{D}_t \tag{B.11}$$

$$\boldsymbol{Y}_t = y\left(\boldsymbol{v}_{t+1}^{e,t}, \boldsymbol{X}_t\right)' \boldsymbol{D}_t, \tag{B.12}$$

where $v_{T+1}^{e,t} = v$ and $X_{T+1}^{e,t} = X$ for all t. The rational expectations are captured by $\mathcal{J}$ and the Jacobian with respect to $d\boldsymbol{X}_s$ is still $\mathcal{J}_{t-s,0}$ for $t \geq s$.

Now, the main observation is that, when considering a partial change $dX_s^{e,\tau}$, nothing changes for $t < \tau$ and so

$$\frac{\partial Y_t}{\partial X_s^{e,\tau}} = \frac{\partial Y_{t-\tau}}{\partial X_{s-\tau}^{e,0}}. \tag{B.13}$$

Note that if we change $X_s^{e,0}$, then $v_s^{e,t} = v$, $y_t = y$, $\Lambda_t = \Lambda$ for $t > 0$. Then,

$$dY_0 = dy_0' D = \left(dy_0^{\text{ann}}\right)' D$$

$$dD_1 = \left(d\Lambda_0^{\text{ann}}\right)' D = dD_1^{\text{ann}}$$

$$dD_{t+1} = \Lambda' dD_t = \left(\Lambda'\right)^t dD_1 = \left(\Lambda'\right)^t dD_1^{\text{ann}}$$

$$dY_t = y' dD_t$$

Note that dY_0 is exactly the same as under rational expectations. For $t \leq s$, we have that

$$dD_t = \left(\Lambda'\right)^{t-1} dD_1^{\text{ann}}$$

$$dY_t = y' dD_t.$$

Futhermore, define the FIRE response as

$$dD_t^{\text{ann}} = (d\Lambda_{t-1}^{\text{ann}})' D + \Lambda' dD_{t-1}^{\text{ann}} = \sum_{m=0}^{t-2} \left(\Lambda'\right)^m (d\Lambda_{t-1-m}^{\text{ann}})' D + \left(\Lambda'\right)^{t-1} dD_1^{\text{ann}}$$

$$dY_t^{\text{ann}} = (dy_t^{\text{ann}})' D + y' dD_t^{\text{ann}}$$

It is useful to put superscripts for the time of the shock, s. For example, define the FIRE response as:

$$dD_t^{*,s} = \sum_{m=0}^{t-2} \left(\Lambda'\right)^m \left(d\Lambda_{t-1-m}^{*,s}\right)' D + \left(\Lambda'\right)^{t-1} dD_1^{*,s}$$

$$dY_t^{*,s} = (dy_t^{*,s})' D + y' dD_t^{*,s}$$

Now, note that

$$dD_t^s = dD_t^{*,s} - \sum_{m=0}^{t-2} \left(\Lambda'\right)^m \left(d\Lambda_{t-1-m}^{*,s}\right)' D$$

$$dY_t = dY_t^{\text{ann}} - (dy_t^{*,s})' D + y' \left(dD_t^s - dD_t^{*,s}\right).$$

Note furthermore, that

$$\begin{aligned}
dD_t^{*,s-1} &= \left(d\Lambda_{t-1}^{*,s-1}\right)' D + \Lambda' dD_{t-1}^{*,s-1} \\
&= \left(d\Lambda_{t-1}^{*,s-1}\right)' D + \Lambda' \left(d\Lambda_{t-1}^{*,s-1}\right)' D + \left(\Lambda'\right)^2 dD_{t-2}^{*,s-1} \\
&= \sum_{m=0}^{t-1} \left(\Lambda'\right)^m \left(d\Lambda_{t-1-m}^{*,s-1}\right)' D + \left(\Lambda'\right)^t \underbrace{dD_0^{*,s-1}}_{=0} \\
&= \sum_{m=0}^{t-1} \left(\Lambda'\right)^m \left(d\Lambda_{t-1-m}^{*,s-1}\right)' D
\end{aligned}$$

and now using the fact that

$$d\Lambda_{t-1-m}^{*,s-1} = d\Lambda_{t-m}^{*,s}$$

we can write

$$dD_t^{*,s-1} = \sum_{m=0}^{t-1} \left(\Lambda'\right)^m \left(d\Lambda_{t-m}^{*,s}\right)' D$$

$$dD_{t-1}^{*,s-1} = \sum_{m=0}^{t-2} \left(\Lambda'\right)^m \left(d\Lambda_{t-1-m}^{*,s}\right)' D.$$

As a result, $dD_t^s = dD_t^{*,s} - dD_{t-1}^{*,s-1}$. Finally, we can write $dY_t^s = dY_t^{s,*} - (dy_t^{*,s})' D - y' dD_{t-1}^{*,s-1}$ and since $dy_t^{*,s} = dy_{t-1}^{*,s-1}$ then

$$dY_t^s = dY_t^{s,*} - \left(dy_{t-1}^{*,s-1}\right)' D - y' dD_{t-1}^{*,s-1} = dY_t^{s,*} - dY_{t-1}^{s-1,*}.$$

For $t \geq s$, we still find that

$$dD_t = \left(\Lambda'\right)^{t-s-1} dD_{s+1}$$

$$dY_t = y' dD_t$$

while the FIRE response would have been

$$dD_t^{\text{ann}} = \Lambda' dD_{t-1}^{\text{ann}} = \left(\Lambda'\right)^{t-s-1} dD_{s+1}^{\text{ann}}$$

$$dY_t^{\text{ann}} = y' dD_t^{\text{ann}}.$$

Once again, we can write

$$dD_{s+1}^{*,s} = \sum_{m=0}^{s-1} \left(\Lambda'\right)^m \left(d\Lambda_{t-1-m}^{*,s}\right)' D + \left(\Lambda'\right)^s dD_1^{*,s}$$

and

$$dD_{s+1}^{s} = \left(\Lambda'\right)^s dD_1^{*,s}.$$

It follows that

$$dD_{s+1}^{s} - dD_{s+1}^{s,*} = -\sum_{m=0}^{s-1} \left(\Lambda'\right)^m \left(d\Lambda_{t-1-m}^{*,s}\right)' D = -dD_s^{*,s-1}$$

and, as a result,

$$dY_t^s = y' dD_t^s = dY_t^{*,s} + y' \left(dD_t^s - dD_t^{*,s}\right) = dY_t^{*,s} + y' \left(\Lambda'\right)^{t-s-1} \left(dD_{s+1}^s - dD_{s+1}^{*,s}\right),$$

$$dY_t^s = dY_t^{*,s} - y' \left(\Lambda'\right)^{t-s-1} dD_s^{*,s-1}$$

and

$$dY_t^s = dY_t^{*,s} - dY_{t-1}^{*,s-1}.$$

It thus follows that

$$\frac{\partial Y_t}{\partial X_s^{e,0}} = \mathcal{J}_{t,s} - \mathcal{J}_{t-1,s-1}. \tag{B.14}$$

Putting everything together We have thus found that

$$\frac{\partial Y_t}{\partial X_s} = \begin{cases} 0 & \text{if } t < s \\ \mathcal{J}_{t-s,0} & \text{if } t \geq s \end{cases}$$

and

$$\frac{\partial Y_t}{\partial X_s^{e,\tau}} = \begin{cases} 0 & \text{if } t < \tau \text{ or } s \leq \tau \\ \mathcal{J}_{t-\tau,s-\tau} - \mathcal{J}_{t-\tau-1,s-\tau-1} & \text{if } t > \tau \text{ and } s > \tau \\ \mathcal{J}_{0,s-t} & \text{if } t = \tau \text{ and } s > \tau = t. \end{cases}$$

Putting everything together we can write

$$dY_t = \sum_{s=0}^{t} \mathcal{J}_{t-s,0} \cdot dX_s + \sum_{\tau=0}^{t-1} \sum_{s=\tau+1}^{\infty} \left(\mathcal{J}_{t-\tau,s-\tau} - \mathcal{J}_{t-\tau-1,s-\tau-1} \right) \cdot dX_s^{e,\tau} + \sum_{s=t+1}^{\infty} \mathcal{J}_{0,s-t} \cdot dX_s^{e,t} \tag{B.15}$$

$$= \sum_{\tau=1}^{t} \sum_{s=\tau}^{\infty} \mathcal{J}_{t-\tau,s-\tau} \left(dX_s^{e,\tau} - dX_s^{e,\tau-1} \right) + \sum_{s=0}^{\infty} \mathcal{J}_{t,s} dX_s^{e,0} \tag{B.16}$$

from where equation (21) follows immediately.

B.3 Special cases

Throughout, we maintain the following notation. $E_t[dX_{t+h}]$ denotes the agent's time-t expectations about the variable at horizon h. $\mathbb{E}_t[dX_{t+h}]$ denotes the full-information and rational expectation for the same variable.

B.3.1 Shallow reasoning

(Angeletos and Sastry, 2021). $E_t[dX_{t+h}] = \lambda \cdot dX_{t+h}$.

$$\Lambda_0 = \begin{bmatrix} 1 & 0 & 0 & 0 & \dots \\ 0 & \lambda & 0 & 0 & \dots \\ 0 & 0 & \lambda & 0 & \dots \\ 0 & 0 & 0 & \lambda & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix} \quad \Lambda_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & \dots \\ 0 & 1 & 0 & 0 & \dots \\ 0 & 0 & \lambda & 0 & \dots \\ 0 & 0 & 0 & \lambda & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix} \tag{B.17}$$

B.3.2 Cognitive discounting

(Gabaix, 2020). $E_t[dX_{t+h}] = \lambda^h \cdot dX_{t+h}$.

$$\Lambda_0 = \begin{bmatrix} 1 & 0 & 0 & 0 & \dots \\ 0 & \lambda & 0 & 0 & \dots \\ 0 & 0 & \lambda^2 & 0 & \dots \\ 0 & 0 & 0 & \lambda^3 & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix} \quad \Lambda_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & \dots \\ 0 & 1 & 0 & 0 & \dots \\ 0 & 0 & \lambda & 0 & \dots \\ 0 & 0 & 0 & \lambda^2 & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix} \tag{B.18}$$

B.3.3 Sticky expectations

(Mankiw and Reis, 2002, Carroll, Crawley, Slacalek, Tokuoka and White, 2018). A date-0 shock ϵ causes a sequence of disturbances $\{dX_t\}$. At each date $t \geq 0$, some agents learn about ϵ and deduce $\{dX_{t+h}\}$ for all $h \geq 0$. The probability of learning ϵ is $1 - \lambda$ for every agent who hasn't learned it already. Thus the share of ignorant agents at date t is λ^{t+1}. They believe that the disturbances observed so far were special events, and don't expect any disturbances in the future. This setup implies that average expectations are $E_t[dX_{t+h}] = (1-\lambda^{t+1}) \cdot dX_{t+h}$.

$$\Lambda_0 = \begin{bmatrix} 1 & 0 & 0 & 0 & \dots \\ 0 & 1-\lambda & 0 & 0 & \dots \\ 0 & 0 & 1-\lambda & 0 & \dots \\ 0 & 0 & 0 & 1-\lambda & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix} \qquad \Lambda_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & \dots \\ 0 & 1 & 0 & 0 & \dots \\ 0 & 0 & 1-\lambda^2 & 0 & \dots \\ 0 & 0 & 0 & 1-\lambda^2 & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix} \tag{B.19}$$

B.3.4 Noisy information and rational expectations

(Angeletos and Huo, 2021). A date-0 shock ϵ causes a sequence of disturbances $\{dX_t\}$ according to an MA process

$$dX_t = M_t \epsilon \tag{B.20}$$

Suppose that agents know the MA coefficients, M_t, but they don't observe ϵ. Their prior is that ϵ is distributed $\mathcal{N}(0, 1/\tau_\epsilon)$. At each date $t \geq 0$, agents receive independent private signals $\epsilon + \nu_t$, where $\nu_t \sim \mathcal{N}(0, 1/\tau_\nu)$. Bayesian updating implies that the average posterior belief is

$$E_t[\epsilon] = \frac{t+1}{\tau_\epsilon/\tau_\nu + t + 1}\epsilon \tag{B.21}$$

Then, the average expectation of dX_{t+h} at date t is

$$E_t[dX_{t+h}] = M_{t+h} E_t[\epsilon] = M_{t+h} \underbrace{\left(\frac{t+1}{\tau_\epsilon/\tau_\nu + t + 1} \right)}_{\lambda_t} \epsilon = \lambda_t dX_{t+h} \tag{B.22}$$

Thus the associated Λ_t matrices are

$$\Lambda_0 = \begin{bmatrix} 1 & 0 & 0 & 0 & \dots \\ 0 & \lambda_0 & 0 & 0 & \dots \\ 0 & 0 & \lambda_0 & 0 & \dots \\ 0 & 0 & 0 & \lambda_0 & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix} \qquad \Lambda_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & \dots \\ 0 & 1 & 0 & 0 & \dots \\ 0 & 0 & \lambda_1 & 0 & \dots \\ 0 & 0 & 0 & \lambda_1 & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix} \tag{B.23}$$

B.3.5 Extrapolation

Geometric extrapolation. $E_t[dX_{t+h}] = \lambda^h dX_t$. First example of non-diagonal Λ matrices.

$$\Lambda_0 = \begin{bmatrix} 1 & 0 & 0 & 0 & \dots \\ \lambda & 0 & 0 & 0 & \dots \\ \lambda^2 & 0 & 0 & 0 & \dots \\ \lambda^3 & 0 & 0 & 0 & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix} \qquad \Lambda_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & \dots \\ 0 & 1 & 0 & 0 & \dots \\ 0 & \lambda & 0 & 0 & \dots \\ 0 & \lambda^2 & 0 & 0 & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix} \tag{B.24}$$

B.3.6 Adaptive expectations

(Cagan, 1956, Friedman, 1957). $E_t[dX_{t+h}] = \lambda^h \kappa \sum_{\tau=0}^{\infty} \lambda^\tau dX_{t-\tau}$, where $\kappa > 0$ scales the geometric sum.

$$\Lambda_0 = \kappa \begin{bmatrix} 1 & 0 & 0 & 0 & \dots \\ \lambda & 0 & 0 & 0 & \dots \\ \lambda^2 & 0 & 0 & 0 & \dots \\ \lambda^3 & 0 & 0 & 0 & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix} \qquad \Lambda_1 = \kappa \begin{bmatrix} 1 & 0 & 0 & 0 & \dots \\ 0 & 1 & 0 & 0 & \dots \\ \lambda^2 & \lambda & 0 & 0 & \dots \\ \lambda^3 & \lambda^2 & 0 & 0 & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix} \tag{B.25}$$

B.3.7 Diagnostic expectations

(Bordalo, Gennaioli and Shleifer, 2018, Bianchi, Ilut and Saijo, 2021). Let $E_t^r[dX_{t+h}]$ denote a reference expectation for the variable h periods ahead. Then, the diagnostic expectation with parameter θ is given by:

$$E_t[dX_{t+h}] = \mathbb{E}_t[dX_{t+h}] + \theta \left(\mathbb{E}_t[dX_{t+h}] - E_t^r[dX_{t+h}] \right).$$

Bordalo et al. (2018) assume that $E_t^r[dX_{t+h}] = \mathbb{E}_{t-1}[dX_{t+h}]$. In this case,

$$\Lambda_0 = \begin{bmatrix} 1 & 0 & 0 & 0 & \dots \\ 0 & 1+\theta & 0 & 0 & \dots \\ 0 & 0 & 1+\theta & 0 & \dots \\ 0 & 0 & 0 & 1+\theta & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}, \quad \Lambda_t = \begin{bmatrix} 1 & 0 & 0 & 0 & \dots \\ 0 & 1 & 0 & 0 & \dots \\ 0 & 0 & 1 & 0 & \dots \\ 0 & 0 & 0 & 1 & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix} \tag{B.26}$$

for $t \geq 1$.

Bianchi et al. (2021) develop a generalization of this framework to allow for long memory, which assumes that $E_t^r[dX_{t+h}] = \sum_{j=1}^{\infty} \alpha_j \mathbb{E}_{t-j}[dX_{t+h}]$. With this assumption, we find

$$\Lambda_0 = \begin{bmatrix} 1 & 0 & 0 & 0 & \dots \\ 0 & 1+\theta & 0 & 0 & \dots \\ 0 & 0 & 1+\theta & 0 & \dots \\ 0 & 0 & 0 & 1+\theta & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}, \quad \Lambda_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & \dots \\ 0 & 1 & 0 & 0 & \dots \\ 0 & 0 & 1+\theta(1-\alpha_1) & 0 & \dots \\ 0 & 0 & 0 & 1+\theta(1-\alpha_1) & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}, \tag{B.27}$$

and, for any t,

$$\Lambda_t = \begin{bmatrix} 1 & 0 & 0 & 0 & \dots \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 1+\theta(1-\sum_{j=1}^{t} \alpha_j) & 0 & \dots \\ 0 & 0 & 0 & 1+\theta(1-\sum_{j=1}^{t} \alpha_j) & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}, \tag{B.28}$$

B.3.8 Noisy information and diagnostic expectations

(Bordalo, Gennaioli, Ma and Shleifer, 2020) As in noisy information and rational expectations, the agent observes a signal $\epsilon + \nu_t$, where $\nu_t \sim \mathcal{N}(0, 1/\tau_\nu)$. However, the forecaster then overweighs representative states by using the distorted posterior

$$f^\theta(\epsilon|S_t^i) = f(\epsilon|S_t^i) R_t^i(\epsilon)^\theta \frac{1}{Z_t} \tag{B.29}$$

Bordalo et al. (2020) assume that $R_t^i(\epsilon) = f(\epsilon|S_t^i)/f(\epsilon|S_{t-1}^i \cup \{\mathbb{E}_{i,t-1}[\epsilon]\})$. This assumption implies that the mean of the distorted posterior is given by:

$$E_{i,t}^\theta[\epsilon] = \mathbb{E}_{i,t}[\epsilon] + \theta\left(\mathbb{E}_{i,t}[\epsilon] - \mathbb{E}_{i,t-1}[\epsilon]\right) \tag{B.30}$$

where $\mathbb{E}_{i,t}[\epsilon]$ denotes the time-t rational expectation with information set S_t^i. It follows that the average expectation is given by

$$\overline{E}_t^\theta[\epsilon] = \underbrace{\left[\frac{\left(\frac{t+1+\theta}{t+1}\right)\tau_\epsilon/\tau_\nu + t}{\tau_\epsilon/\tau_\nu + t}\right]\frac{t+1}{\tau_\epsilon/\tau_\nu + t + 1}}_{\equiv \lambda_t}\epsilon \tag{B.31}$$

Thus the associated Λ_t matrices are

$$\Lambda_0 = \begin{bmatrix} 1 & 0 & 0 & 0 & \dots \\ 0 & \lambda_0 & 0 & 0 & \dots \\ 0 & 0 & \lambda_0 & 0 & \dots \\ 0 & 0 & 0 & \lambda_0 & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}, \quad \Lambda_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & \dots \\ 0 & 1 & 0 & 0 & \dots \\ 0 & 0 & \lambda_1 & 0 & \dots \\ 0 & 0 & 0 & \lambda_1 & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}. \tag{B.32}$$

Analogously to Bianchi et al. (2021), we can extend this model to include long-memory as follows. Assume that $R_t^\theta(\epsilon) = f(\epsilon|S_t^i)/f^*(\epsilon|S_t^i)$, where $\epsilon \sim_{f^*|S_t^i} \mathcal{N}(E_t^r[\epsilon], \tau_\epsilon + (t+1)\tau_\nu)$ and $E_t^r[\epsilon] = \sum_{j=1}^t \alpha_j \mathbb{E}_{i,t-j}[\epsilon]$. It follows that

$$E_{i,t}^\theta[\epsilon] = \mathbb{E}_{i,t}[\epsilon] + \theta\left(\mathbb{E}_{i,t}[\epsilon] - E_{i,t}^r[\epsilon]\right). \tag{B.33}$$

As a result, the average expectation is given by

$$\overline{E}_t^\theta[\epsilon] = \left[(1+\theta)\frac{t+1}{\tau_\epsilon/\tau_\nu + t + 1} - \theta\sum_{j=1}^t \alpha_j\left(\frac{t+1-j}{\tau_\epsilon/\tau_\nu + t + 1 - j}\right)\right]\epsilon, \tag{B.34}$$

and defining now $\lambda_t \equiv (1+\theta)\frac{t+1}{\tau_\epsilon/\tau_\nu+t+1} - \theta\sum_{j=1}^t \alpha_j\left(\frac{t+1-j}{\tau_\epsilon/\tau_\nu+t+1-j}\right)$ we obtain the analogous Λ_t matrices as above.

C Appendix to section 4

C.1 Financial intermediary

Set up decision problem formally and derive no-arbitrage conditions.

C.2 Retailers

The Bellman equation of firm j is

$$J_t(p_{jt-1}) = \max_{k_{jt},l_{jt},y_{jt},p_{jt}} \left\{ \frac{p_{jt}}{P_t} y_{jt} - h_t l_{jt} - r_t^K k_{jt} - \frac{\psi_p}{2} \left[\log\left(\frac{p_{jt}}{p_{jt-1}} \right) \right]^2 Y_t + \frac{J_{t+1}(p_{jt})}{1+r_t^e} \right\}$$
$$\text{s.t. } y_{jt} = F_t(k_{jt}, l_{jt})$$
$$y_{jt} = \left(\frac{p_{jt}}{P_t} \right)^{-\epsilon} Y_t$$

Substitute the production function and write the problem as

$$J_t(p_{jt-1}) = \max_{k_{jt},l_{jt},p_{jt}} \left\{ \frac{p_{jt}}{P_t} F_t(k_{jt}, l_{jt}) - h_t l_{jt} - r_t^K k_{jt} - \frac{\psi_p}{2} \left[\log\left(\frac{p_{jt}}{p_{jt-1}} \right) \right]^2 Y_t + \frac{J_{t+1}(k_{jt}, p_{jt})}{1+r_t^e} \right\}$$
$$\text{s.t. } \frac{p_{jt}}{P_t} Y_t = \left(\frac{F_t(k_{jt}, l_{jt})}{Y_t} \right)^{-\frac{1}{\epsilon}} Y_t$$

Let η_{jt} denote the Lagrange multiplier on the constraint. The FOCs with respect to p_{jt} and p_{jt-1} are

$$0 = \frac{1}{P_t} F_t(k_{jt}, l_{jt}) - \psi_p \log\left(\frac{p_{jt}}{p_{jt-1}} \right) \frac{Y_t}{p_{jt}} - \eta_{jt} \frac{Y_t}{p_{jt}} + \frac{\partial_p J_{t+1}(k_{jt}, p_{jt})}{1+r_t^e} \tag{C.1}$$

$$\partial_p J_t(k_{jt-1}, p_{jt-1}) = \psi_p \log\left(\frac{p_{jt}}{p_{jt-1}} \right) \frac{Y_t}{p_{jt-1}} \tag{C.2}$$

In symmetric equilibrium, the FOCs simplify to

$$0 = \frac{1}{P_t} F(u_t k_{t-1}, L_t) - \psi_p \log\left(\frac{P_t}{P_{t-1}} \right) \frac{Y_t}{P_t} - \eta_t \frac{Y_t}{P_t} + \frac{1}{1+r_t^e} \psi_p \log\left(\frac{P_{t+1}}{P_t} \right) \frac{Y_{t+1}}{P_t} \tag{C.3}$$

$$0 = Y_t - \psi_p \log\left(\frac{P_t}{P_{t-1}} \right) Y_t - \eta_t Y_t + \frac{1}{1+r_t^e} \psi_p \log\left(\frac{P_{t+1}}{P_t} \right) Y_{t+1} \tag{C.4}$$

$$\log(1+\pi_t) = \frac{1}{\psi_p}(1-\eta_t) + \frac{1}{1+r_t^e} \frac{Y_{t+1}}{Y_t} \log(1+\pi_{t+1}) \tag{C.5}$$

where $\pi_t = P_t/P_{t-1} - 1$ is the inflation rate. Define the real marginal cost as $mc_t \equiv (\epsilon - \eta_t)/\epsilon$. Then the equilibrium conditions can be summarized as

- Phillips curve:

$$\log\left(1+\pi_t\right) = \frac{\psi_p}{\epsilon}\left(mc_t - \frac{\epsilon-1}{\epsilon}\right) + \frac{1}{1+r_t^e}\frac{Y_{t+1}}{Y_t}\log\left(1+\pi_{t+1}\right) \tag{C.6}$$

- Labor demand:

$$h_t = mc_t \cdot \partial F_L(\tilde{K}_t, L_t) = mc_t(1-\alpha)\frac{Y_t}{L_t} \tag{C.7}$$

- Capital demand:

$$r_t^K = mc_t \cdot \partial F_K(\tilde{K}_t, L_t) = mc_t\alpha\frac{Y_t}{\tilde{K}_t} \tag{C.8}$$

- Production:

$$Y_t = F_t(\tilde{K}_t, L_t) = \Theta_t \tilde{K}_t^{\alpha} L_t^{1-\alpha} \tag{C.9}$$

- Price adjustment cost:

$$\Psi_t = \frac{\psi_p}{2}\Big[\log\left(1+\pi_t\right)\Big]^2 Y_t \tag{C.10}$$

- Dividends:

$$d_t^R = Y_t - h_t L_t - r_t^K \tilde{K}_t - \Psi_t \tag{C.11}$$

C.3 Capital producer

The Bellman equation is

$$\begin{aligned} J_t(K_{t-1}, I_{t-1}) = \max_{K_t, I_t} &\left\{ r_t^K K_{t-1} - I_t + \frac{J_{t+1}(K_t, I_t)}{1+r_t} \right\} \\ \text{s.t. } K_t = (1-\delta)K_{t-1} &+ \mu_t\left[1 - S\left(\frac{I_t}{I_{t-1}}\right)\right] I_t \end{aligned} \tag{C.12}$$

Let's define Tobin's Q as the marginal value of capital at the end of period t

$$Q_t \equiv \frac{\partial_K J_{t+1}(K_t, I_t)}{1+r_t} \tag{C.13}$$

The FOC with respect to K_{t-1} is

$$\partial_K J_t(K_{t-1}, I_{t-1}) = r_t^K + \frac{\partial_K J_{t+1}^K(K_t, I_t)}{1+r_t}\left(1-\delta\right) \tag{C.14}$$

$$Q_t(1+r_t) = r_{t+1}^K + Q_{t+1}\left(1-\delta\right) \tag{C.15}$$

The FOC with respect to I_{t-1} is

$$\partial_I J_t(K_{t-1}, I_{t-1}) = \mu_t Q_t \left(\frac{I_t}{I_{t-1}}\right)^2 S'\left(\frac{I_t}{I_{t-1}}\right) \tag{C.16}$$

The FOC with respect to I_t is

$$0 = -1 + Q_t \mu_t \left[1 - S\left(\frac{I_t}{I_{t-1}}\right) - \left(\frac{I_t}{I_{t-1}}\right) S'\left(\frac{I_t}{I_{t-1}}\right)\right] + \frac{\partial_I J_{t+1}}{1 + r_t^e} \tag{C.17}$$

To summarize, the equilibrium conditions of the capital producer are

- Valuation:

$$1 + r_t = \frac{r_{t+1}^K + Q_{t+1}(1-\delta)}{Q_t} \tag{C.18}$$

- Investment:

$$1 = Q_t \mu_t \left[1 - S\left(\frac{I_t}{I_{t-1}}\right) - \left(\frac{I_t}{I_{t-1}}\right) S'\left(\frac{I_t}{I_{t-1}}\right)\right] + \frac{\mu_{t+1} Q_{t+1}}{1 + r_t^e} \left(\frac{I_{t+1}}{I_t}\right)^2 S'\left(\frac{I_{t+1}}{I_t}\right) \tag{C.19}$$

- Capital law of motion:

$$K_t = (1-\delta) K_{t-1} + \mu_t \left[1 - S\left(\frac{I_t}{I_{t-1}}\right)\right] I_t \tag{C.20}$$

- Dividends:

$$d_t^K = r_t^K K_{t-1} - I_t \tag{C.21}$$

For concreteness, let the $S(\bullet)$ be quadratic

$$S\left(\frac{I_t}{I_{t-1}}\right) = \frac{\psi}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2 \tag{C.22}$$

$$S'\left(\frac{I_t}{I_{t-1}}\right) = \psi\left(\frac{I_t}{I_{t-1}} - 1\right) \tag{C.23}$$

C.4 Labor agency

The Bellman equation is

$$\begin{aligned} J_t(N_{t-1}) &= \max_{N_t, v_t} \left\{ (h_t - w_t) N_t - (\kappa_v + \kappa_h q_t) v_t + \frac{J_{t+1}(N_t)}{1 + r_t} \right\} \\ &\text{s.t. } N_t = (1 - s_t) N_{t-1} + q_t v_t \end{aligned} \tag{C.24}$$

Let λ_t denote the Lagrange multiplier on the constraint. The FOCs wrt N_t, v_t, and N_{t-1} are

$$0 = h_t - w_t - \lambda_t + \frac{J'_{t+1}(N_t)}{1 + r_t} \tag{C.25}$$

$$0 = -\kappa_v - \kappa_h q_t + \lambda_t q_t \tag{C.26}$$

$$J'_t(N_{t-1}) = \lambda_t (1 - s_t) \tag{C.27}$$

Combining these yields the job creation curve. In sum, the equilibrium conditions are

- Job creation:

$$\frac{\kappa_v}{q_t} + \kappa_h = h_t - w_t + \frac{1 - s_{t+1}}{1 + r_t} \left(\frac{\kappa}{q_{t+1}} + \kappa_h \right) \tag{C.28}$$

- Dividends:

$$d_t^L = (h_t - w_t)N_t - (\kappa_v + \kappa_h q_t)v_t \tag{C.29}$$

D Appendix to Section 5

D.1 Fitting a model of beliefs

Figure D.1: Illustration of parametric belief models

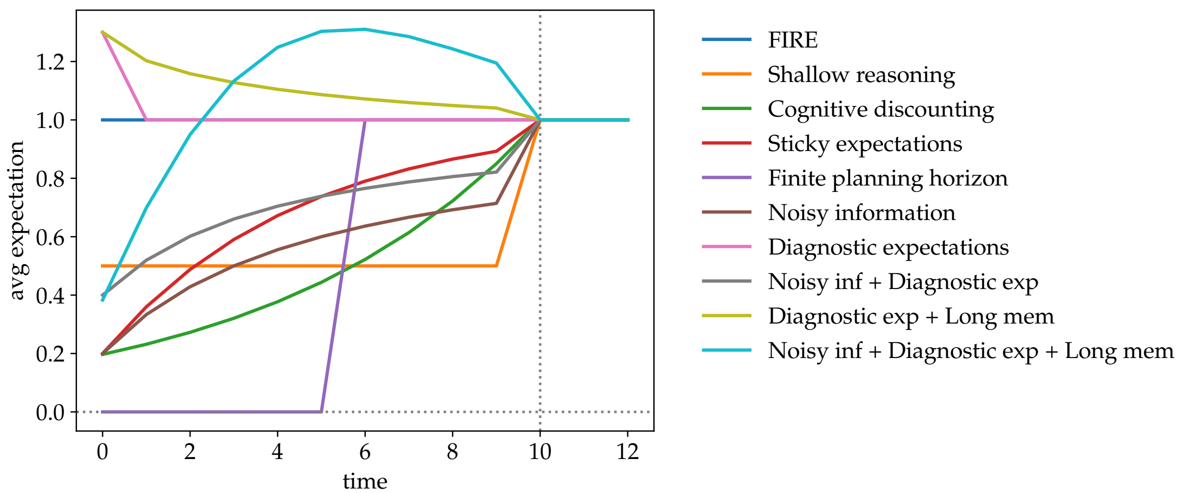
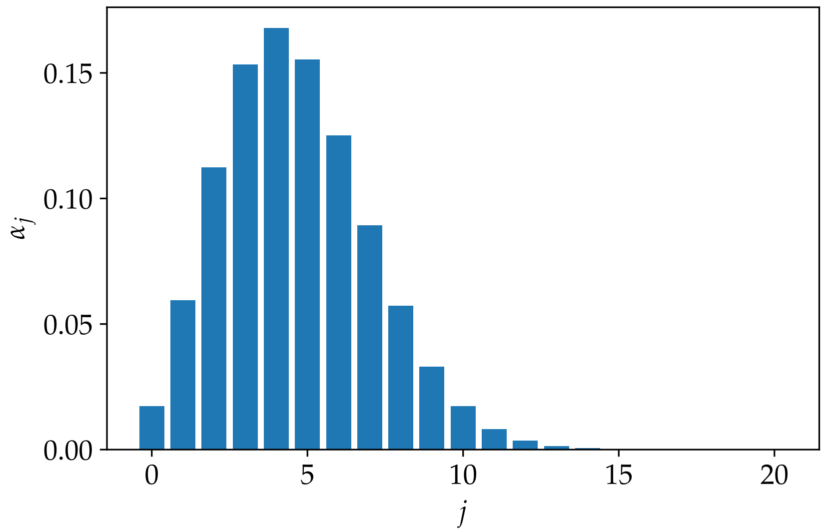


Figure D.2: Estimated memory weights



D.2 Decomposing GE forces

Figure D.3 shows the decomposition of the aggregate consumption response into all the variables that enter the aggregate consumption function directly for each model of belief formation used in our analysis. The job-finding rate and UI duration emerge as the main drivers of consumption response in the first four quarters. The shape of the consumption response to the UI duration is recognizable from the partial equilibrium exercise.

Figure D.3: Decomposition of general equilibrium consumption response